

# The Analysis of Multiple Underream Anchors

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## 1 INTRODUCTION

In recent years, there has been an increasing acceptance of the use of multiple underream anchor systems for the support of both temporary and permanent structures. This acceptance has been facilitated by the development of construction techniques which allow convenient and economical installation of multiple underreams, and has been accompanied by the formulation of empirical rules for use in design. Some experimental research has been undertaken into the performance of multiple underream anchors (e.g. Swain, 1976) however there has been relatively little theoretical research into the factors influencing the behaviour of these anchors.

In this paper, an analytical technique for calculating the elastic response of multiple underream anchors is outlined and is then used to investigate the effects of the number and spacing of underreams upon the behaviour of anchor systems resting in an elastic half-space. In this study, consideration is given to the influence of the anchor's proximity to the free surface and its inclination upon the elastic response.

The results of this study into multiple underream anchor behaviour are summarised in the form of influence charts which may be used in hand calculations for estimating the elastic response of anchor systems for a wide range of anchor depths, inclinations, number and spacing of underreams.

It is considered that the elastic solutions presented in this paper for multiple underream anchors, may be used for designing anchor systems in the same manner that elastic solutions are currently used for designing surface footings and pile foundations.

## 2 THEORY

Figure 1 shows a series of anchors  $A_1, \dots, A_n$  in an elastic soil. Each anchor  $A_i$  is subdivided<sup>n</sup> into a number of subregions or elements  $D_1, \dots, D_m$  and it is assumed that the forces  $F_{ij} = F_{ij}(X, Y, Z)$ , acting on each element are uniformly distributed over that element. The theory of elasticity may then be used to establish the relation

$$\underline{W} = \underline{J} \underline{F} \quad (1)$$

where  $\underline{W}^T = (\bar{w}_{11}^T, \bar{w}_{12}^T, \dots, \bar{w}_{nm}^T)$

is the vector of element displacements

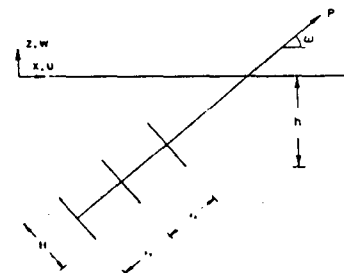
$$\underline{F}^T = (F_{11}^T, F_{12}^T, \dots, F_{nm}^T)$$

is the vector of element forces

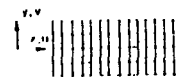
and  $\bar{w}_{ij}$  is the average deflection of "jth" element of the "ith" plate

and  $\underline{J}$  is a  $(3nm) \times (3nm)$  matrix of influence coefficients.

Details regarding the calculation of these influence coefficients are given by Rowe and Booker (1979b).



(a) ELEVATION



(b) PLAN

FIG. 1 TYPICAL PROBLEM CONFIGURATION

If the anchors are rigid, then each anchor will only undergo rigid body movement. In this paper attention will be restricted to the case of inextensible anchor rods and it will be assumed that each anchor undergoes a translation  $\delta$  parallel to the anchor shaft. Clearly this assumption is only correct if the anchor rod is rigidly attached to the anchor shaft, however previous results obtained by Rowe and Booker (1979b, 1979c) indicate that in general very little rotation or deviation of the translation from the direction of the

anchor shaft occurs. Thus in the majority of cases, and particularly for cases of practical interest, this assumption is quite accurate.

Equation (1) may therefore be written

$$\underline{J} \underline{F} = \delta \underline{a} \quad (2)$$

where  $\underline{a}$  is a vector of length  $(3nm)$  as defined below

$$\underline{a}^T = (1_x, 1_y, 1_z, 1_x, 1_y, 1_z \dots)$$

and  $(1_x, 1_y, 1_z)^T$  is a unit vector parallel to the anchor rods.

Suppose that equation 2 is solved for  $\delta = \hat{\delta}$  (conveniently taken to be unity) and that in this case  $\underline{F} = \hat{\underline{F}}$

$$\text{so that} \quad \hat{\underline{F}} = \hat{\delta} \underline{J}^{-1} \underline{a} \quad (3)$$

The tension  $T_k = \hat{T}_k$  in the "kth" rod may now be found from equilibrium since

$$\hat{T}_k = \sum_{i=k}^n \sum_{j=1}^m (1_x \hat{X}_{ij} + 1_y \hat{Y}_{ij} + 1_z \hat{Z}_{ij}) \quad (4)$$

and thus the applied load  $P = \hat{P}$  is given by

$$\hat{P} = \sum_{k=1}^n \sum_{j=1}^m (1_x \hat{X}_{ij} + 1_y \hat{Y}_{ij} + 1_z \hat{Z}_{ij})$$

It now follows from linearity that in the general situation

$$\begin{aligned} \delta &= \frac{P}{\hat{P}} \hat{\delta} \\ \underline{F}_{ij} &= \frac{P}{\hat{P}} \hat{\underline{F}}_{ij} \\ \underline{T}_k &= \frac{P}{\hat{P}} \hat{\underline{T}}_k \end{aligned} \quad (5)$$

### 3 THE BEHAVIOUR OF MULTIPLE ANCHORS

The theory outlined in section 2 may be used to analyse the general case involving arbitrary anchor inclination, number of underreams and spacing between underreams. In this paper, consideration will be restricted to a number of typical cases involving no more than five underreams which are positioned at equal spacings. The solutions were obtained for underreams which are square in section, however a comparison between the elastic response of square and circular anchors (Rowe and Booker, 1979a) indicates that the solutions for a square anchor may be used for circular anchors of equal area. Provision for the effect of anchor shape can be made by modifying Selvadurai's (1976) analytical solution for a rigid circular anchor at infinite depth; viz

$$\delta_{\infty} = \frac{c_{\infty}}{BE} \cdot P \quad (6a)$$

where  $\delta_{\infty}$  is the displacement of an anchor at infi-

nite depth, of diameter (or width B), subjected to an applied load P.

E is the Young's modulus of the soil;

$$c_{\infty} \text{ (circular)} = \frac{(1+\nu)(3-4\nu)}{8(1-\nu)} \quad (6b)$$

$$c_{\infty} \text{ (square)} = \sqrt{\pi} \frac{(1+\nu)(3-4\nu)}{16(1-\nu)} \quad (6c)$$

and  $\nu$  is Poisson's ratio of the soil.

If it is assumed that the anchor rod connecting the underreams is relatively inextensible, then the deflection of the anchor system may be given in terms of the results for a single anchor at infinite depth multiplied by a correction factor  $M_G$ ; i.e:

$$\delta = \delta_{\infty} \cdot M_G = \frac{c_{\infty}}{BE} \cdot M_G \cdot P \quad (7)$$

where the correction factor  $M_G$  incorporates the effects of:

- anchor inclination angle  $\omega$  (see Figure 1);
- the distance h between the soil surface and the bottom of the leading underream;
- the number of underreams;
- the spacing s between underreams.

Although the theory can be generalised to consider a number of different boundary conditions at the interface, attention will be restricted here to the rigid displacement of an underream system with rough anchor plates which are fully bonded to the soil. The case of a fully bonded anchor is considered to be the most practical limiting case for the application of elastic solutions since it has been shown by Rowe (1978) that separation of an anchor plate from the underlying soil is often associated with significant plastic failure within the soil mass; this usually occurs at loads well above the working load.

If interaction effects were neglected, then superposition of the elastic results for a single underream (Rowe and Booker, 1979b) could be used to estimate the displacement of multiple anchor systems. However, one might expect that this approach would lead to an unconservative estimate of anchor displacement since interaction between anchors will reduce the efficiency of the anchor system. To illustrate the magnitude of the interaction between underreams, consider an anchor system with n underreams at very great depth and subjected to a load P. If the displacement of an isolated underream subjected to unit load is  $\delta_0$ , then the displacement of the anchor system (neglecting interaction) would be  $\delta_s = \delta_0 P/n$ . Now if the actual displacement of the anchor system (allowing for interaction) is  $\delta_G$  then the increase in displacement of the anchor system due to interaction between underreams is given by the ratio  $\delta_G/\delta_s$ ; this ratio is shown in Figure 2 for a number of cases. In the case of a 2 underream system, the actual displacement of the anchor

system exceeds the value that would be determined from a simple analysis, neglecting interaction, by more than 10% for anchor spacings less than 6 anchor widths (diameters). The degree of interaction increases with the number of underreams; thus, with a 5 underream system an unpractical spacing of almost 20 anchor plate widths is required between each adjacent pair of underreams, to achieve negligible interaction.

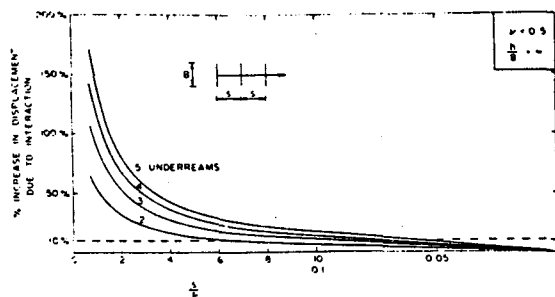


FIG. 2 EFFECT OF INTERACTION UPON THE DISPLACEMENT OF A MULTIPLE UNDERREAM ANCHOR AT INFINITE DEPTH

Figure 2 could be used to modify the displacement obtained by superposition of single underream results to give the correct elastic response for a particular deep anchor system. However it is more convenient to express the displacement of the anchor system in terms of the solution for a single deep underream, multiplied by a correction factor  $M_c$ , as indicated in Equation 7. The correction factor  $M_c$  for an anchor system at great depth ( $h/b \gg \infty$ ) is given in Figure 3 for  $\nu = 0.5$ . From consideration of the results given in Figure 3 it may be concluded that increasing the number of underreams leads to an appreciable decrease in anchor system displacement, although the magnitude of this reduction is dependent upon the spacing between underreams. Indeed the results indicate that it is more beneficial (in terms of reduced anchor displacement for a given load) to increase the number of underreams at the expense of reducing the spacing, provided that the spacing is not reduced to less than 1.5 anchor widths. For example, the deflection of an anchor system with two underreams at a spacing of 8 anchor widths (for a given total load  $P$ ) is more than 50% greater than the displacement of an anchor system with five underreams at a spacing of two anchor widths; there is a monotonic variation in displacement with the number of underreams between these two cases.

It may be anticipated that the response of a multiple underream anchor will be influenced by the depth  $h$  of the anchor system beneath the soil surface and the inclination  $\omega$  of the anchor system to the horizontal. The variation in anchor performance with anchor inclination and the number and spacing of underreams is quite complex. In part, this apparent complexity arises from the manner in which the results are compared in non-dimensional form. For example, the embedment ratio is defined as the depth to the bottom of the leading underream and so altering the inclination of the anchor system changes the minimum distance between the underream and the soil surface while the maximum distance remains constant (for given  $h/b$ ). It is found that the elastic response

(displacement for a given load) of a very shallow (e.g.  $h/b = 1$ ) single underream does not necessarily vary monotonically with inclination angle  $\omega$ , however for most single anchors (i.e.  $h/b > 1.5$ ) the elastic response does increase with the inclination angle. For second and subsequent underreams in the anchor system, both the maximum and minimum distance between the underreams and the surface increase with the inclination angle  $\omega$  (for given  $h/b$  to the leading underream). Consequently, the elastic response of the latter underreams decreases as the anchor system is rotated from  $\omega = 0^\circ$  through to  $\omega = 90^\circ$ . The combination of these two effects results in the trend shown in Figure 4 for  $h/b = 1$ , where the displacement of an anchor system for a particular number and spacing of underreams is greatest when  $\omega = 0^\circ$  and is least for  $\omega = 45^\circ$ . The displacement for a system with  $\omega = 90^\circ$  is slightly greater than that for  $\omega = 45^\circ$ .

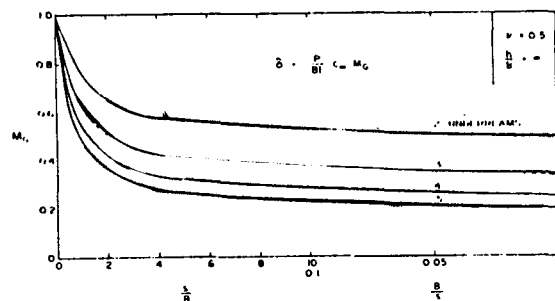


FIG. 3 DISPLACEMENT REDUCTION FACTOR  $M_c$  FOR AN ANCHOR SYSTEM AT INFINITE DEPTH  $\nu = 0.5$

The solution where the anchor system is horizontal  $\omega = 0^\circ$  may be regarded as a special case since the depth of each underream is independent of spacing,  $s$ . For all other cases the depth of second and subsequent underreams increases with increasing spacing between underreams and it is found that for  $\omega > 15^\circ$  the variation in the displacement of a given multiple anchor system with changing inclination angle  $\omega$  is less than 7%. Thus for  $\omega > 15^\circ$  the variation in the elastic response of shallow anchors for different inclinations  $\omega$  decreases with increasing spacing  $s$  between underreams, however because of the special nature of the case where  $\omega = 0^\circ$ , the difference between solutions for  $\omega = 0^\circ$  and those for  $\omega > 0^\circ$  becomes more pronounced at large spacings.

Anchor inclination becomes less important as the embedment ratio becomes larger. This trend may be appreciated by comparing the results given in Figures 4, 5, 6 and 7 for a range of embedment ratios. These results indicate a relatively complex interaction between the effects of inclination, embedment ratio and the number and spacings of underreams. However in practical terms, for anchor systems with  $h/b > 3$  the effect of inclination upon anchor response can be largely neglected since there is less than a 7% variation in anchor system response over the entire range of inclination  $0^\circ < \omega < 90^\circ$ .

The proximity of an anchor system to the free surface has a noticeable effect upon the anchor system response although, in general, this effect is not as significant as the interaction due to the spacing between underreams. Figure 3 to 7 indicate the variation in displacement of multiple

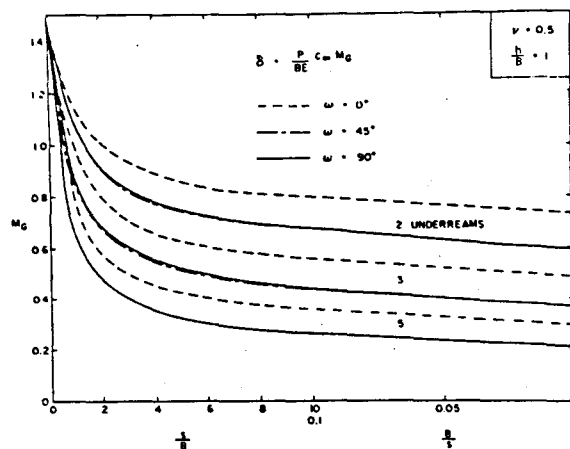


FIG. 4 VARIATION IN  $M_G$  WITH SPACING  $s/B$  AND INCLINATION:  $\frac{h}{B} = 1$ ,  $\nu = 0.5$

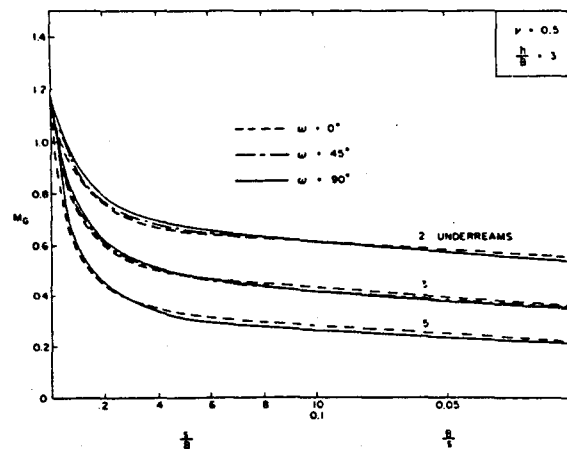


FIG. 5 VARIATION IN  $M_G$  WITH SPACING AND INCLINATION:  $\frac{h}{B} = 3$ ,  $\nu = 0.5$

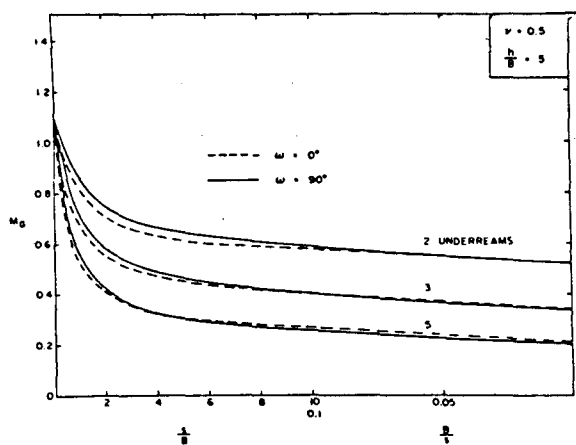


FIG. 6 VARIATION IN  $M_G$  WITH SPACING AND INCLINATION:  $\frac{h}{B} = 5$ ,  $\nu = 0.5$

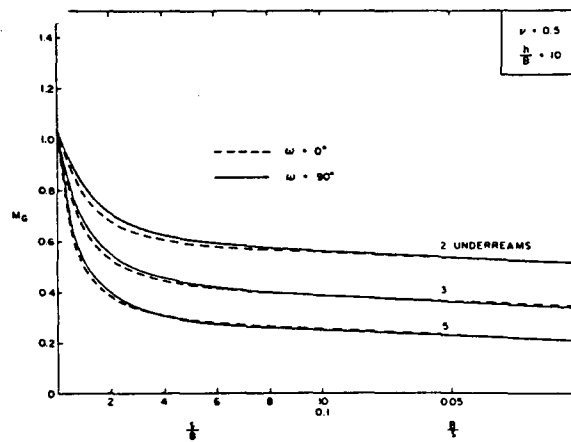


FIG. 7 VARIATION IN  $M_G$  WITH SPACING AND INCLINATION:  $\frac{h}{B} = 10$ ,  $\nu = 0.5$

anchor systems as a function of underream spacing for a range of embedment ratios. A comparison of the results for an anchor system at infinite depth (Figure 3) with anchor systems at finite depth shows two types of behaviour. For anchor systems with inclination  $\omega > 0^\circ$ , the embedment ratio  $h/B$  becomes less important as either the number or spacing of the underreams is increased. This trend is to be expected since increasing either parameter leads to an increase in the distance between the bottom of the anchor system and the soil surface. However for the special case of anchors with inclination  $\omega = 0^\circ$ , there is no change in depth associated with altering the number and spacing of underreams and in this case, the depth of embedment has a more pronounced influence upon anchor performance than is observed when  $\omega > 0^\circ$ .

The effect of proximity to the soil surface decays

rapidly with increasing embedment and the displacement of anchor systems with an embedment ratio of 10 is generally within 10% of that for similar systems at infinite depth. Thus for practical purposes, an anchor system may be considered to be "deep" at an embedment ratio of 10. Under some circumstances, notably for systems with  $\omega = 0^\circ$  and only a few underreams, the critical embedment ratio (i.e. the embedment ratio at which the displacement is within 10% of the value for an infinitely deep anchor) may be as low as 4.

The influence of Poisson's ratio upon anchor response depends upon anchor depth and inclination as well as the number and spacing of underreams. To indicate the effect of Poisson's ratio upon anchor displacements, the ratio of the anchor displacement for a given  $\nu$ , denoted by  $\delta(\nu)$ , to the displacement for  $\nu = 0.5$ , denoted by  $\delta(\nu = 0.5)$ , is shown in

Figures 8 and 9 as a function of  $\nu$  for a number of specific cases. These results raise a number of interesting points. Firstly it will be noted (see Figure 8) that for an isolated anchor at infinite depth the displacement for  $\nu = 0$  is equal to that for  $\nu = 0.5$  and the maximum displacement at  $\nu = 0.3$  is only 11% above that for  $\nu = 0.5$  (this result follows directly from Equation 6.) This situation is markedly different from that encountered with a surface footing where the increase in displacement for  $\nu = 0.3$  is 21% and this rises to 33% for  $\nu = 0$ . The important practical implication of this result for anchors (or footings) in clay is that increasing the embedment of an anchor decreases the amount of "consolidation settlement" that would be obtained, particularly for soils with a low drained Poisson's ratio. Furthermore, with respect to the effect of Poisson's ratio upon anchor behaviour, the transition from surface footing behaviour to deep anchor behaviour is very rapid. All single anchors with an embedment ratio greater than or equal to three show a very similar variation in displacement with Poisson's ratio as that indicated for an infinitely deep anchor. This allows convenient, and relatively accurate, determination of the displacement for an anchor in a soil with any Poisson's ratio  $\nu$  by interpolation of the results given in this paper.

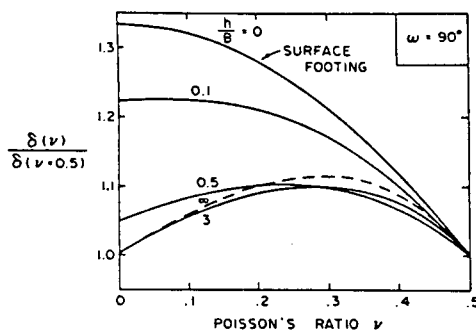


FIG. 8 EFFECT OF POISSON'S RATIO UPON DISPLACEMENT OF A SINGLE ANCHOR PLATE

A similar trend in the effect of Poisson's ratio upon anchor displacement is observed for multiple underream anchors (see Figure 9) although in this case the influence of Poisson's ratio depends not only on anchor depth and inclination, but also upon the number and spacing of underreams. The interaction between underreams is greatest for an incompressible material and decreases with Poisson's ratio; since interaction has its greatest effect upon anchor systems with a large number of closely spaced underreams, it might then be expected that the influence of Poisson's ratio upon the anchor displacement would be greatest for these cases. Indeed Figure 9 demonstrates that under certain circumstances the displacement of a multiple anchor system in an incompressible soil will be larger than the displacement for  $\nu < 0.5$  (assuming the same elastic modulus). Of greater practical importance however is the fact that for a wide range of conditions the displacement of a multiple anchor system is relatively insensitive to the value of Poisson's ratio, generally varying by less than 10% over the entire range of values of  $\nu$ .

The solutions presented in this paper are for an anchor system embedded in an elastic half-space. Provision could be made in the analysis for the case where there is a rigid stratum at finite depth beneath the anchor system, however results obtained by Rowe and Booker (1979a) for a single, horizontally embedded, circular anchor plate indicate that the depth to the rigid stratum has relatively little effect (i.e. less than 10%) upon single anchor response provided the rigid stratum is at least 10 anchor plate diameters below the anchor. Accordingly it is considered that solutions obtained for a half-space are sufficiently accurate for practical cases where the soil mass extends at least 10 anchor diameters below the deepest underream; the displacement obtained using half-space theory will be conservative.

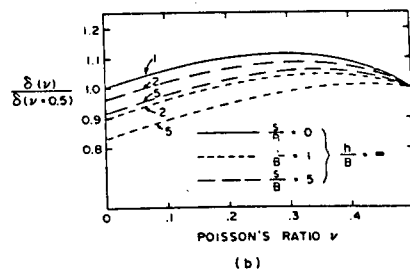
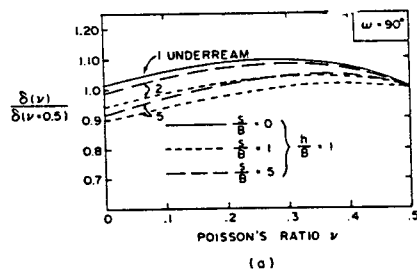


FIG. 9 EFFECT OF POISSON'S RATIO UPON ANCHOR DISPLACEMENT

#### 4

#### CONCLUSIONS

The behaviour of multiple underream anchor systems, resting in a homogeneous elastic half-space has been examined using an analytical technique which is outlined in the paper. The study has shown that in general, the elastic performance of an anchor system is significantly enhanced by increasing the number of underreams, even at the expense of a reduction in spacing between the underreams.

Consideration of anchor systems with a given number of underreams indicated that anchor spacing was the dominant parameter influencing the anchor system response. The effect of interaction between underreams upon anchor behaviour increased with the number of underreams, however even for small anchor systems, significant interaction may occur at relatively large spacings. Consequently, it is usually necessary to take account of interaction effects when predicting anchor behaviour.

The depth of anchor system embedment noticeably influenced the response of shallow anchor systems,

however the influence of the free surface decayed rapidly with increasing embedment and for practical purposes may be neglected when the depth of soil above the anchor system is greater than ten underream diameters.

Anchor inclination was generally found to have very little effect upon anchor system response and in particular it was found that it is usually sufficient to restrict attention to the limiting cases of anchor systems with vertical and horizontal axes.

Finally, it was shown that the elastic response of a multiple anchor system was relatively insensitive to the value of Poisson's ratio and it is suggested that anchor systems in clay would exhibit relatively little consolidation displacement.

Parametric solutions are presented in the form of influence charts which may be used directly in hand calculations to predict the elastic load deflection behaviour of multiple underream anchors for a wide range of parameters; the use of the influence charts is illustrated by a worked example in the appendix.

#### 5 ACKNOWLEDGEMENT

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#### 7 APPENDIX ILLUSTRATIVE EXAMPLE

Estimate the elastic displacement of the anchor system shown in Figure 10 for a soil with  $E' = 3000$  kPa,  $\nu = 0.4$  where the anchor system is subjected to a working load of 1000 kN.

The non-dimensional parameters are

$$h/B = \frac{3}{0.54} = 5.5 \quad ; \quad s/B = \frac{1.15}{0.54} = 2.1$$

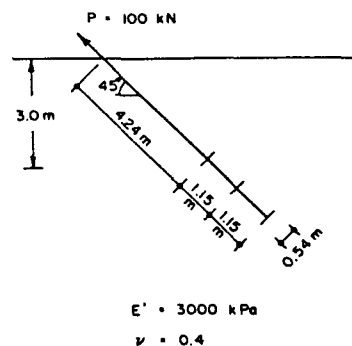


FIG. 10 EXAMPLE

From Equation 7

$$\delta = \frac{c_{\infty}}{BE} M_G P$$

where

$$c_{\infty} = \frac{(1+\nu)(3-4\nu)}{8(1-\nu)} = 0.408 \text{ for } \nu = 0.4$$

$$= 0.375 \quad \nu = 0.5$$

since the curves for  $\nu = 0.4$  are not given, use  $M_G$  results for  $\nu = 0.5$  and correct by interpolation from Figure 9.

(1) From Figure 6 for a 3 underream system,

$$h/B = 5, \quad s/B = 2.1, \quad M_G = 0.565$$

Now determine

$$\delta(\nu = 0.5) = \frac{c_{\infty}(\nu = 0.5) \cdot M_G(\nu = 0.5) P}{B.E.}$$

$$= \frac{0.375 \times 0.565 \times 100}{0.54 \times 3000}$$

$$= 0.0130$$

(2) Estimate  $\frac{\delta(\nu)}{\delta(\nu = 0.5)}$  by interpolation from Figure 9

$$\text{Thus } \frac{\delta(\nu)}{\delta(\nu = 0.5)} = 1.04 \text{ for } s/B = 2, 3 \text{ underreams}$$

(3)  $(\nu = 0.4) = 1.04 \times 0.0130 = 0.0136\text{m}$

i.e. 13.6 mm

This is 6.6% above the displacement of 12.8 mm obtained from a full analysis. Note that the approximate answer was determined using the results for  $h/B = 5$ ; an additional refinement would be to interpolate between the results for  $h/B = 5$ ,  $h/B = 10$  to obtain  $M_G$  for  $h/B = 5.5$ .