

THE ELASTIC RESPONSE OF MULTIPLE UNDERREAM ANCHORS

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SUMMARY

A technique is developed for the analysis of multiple underream anchor systems resting in an elastic soil. This technique may be used to consider anchor systems involving arbitrary anchor inclination and depth beneath the soil surface, as well as arbitrary number, shape, size and spacing of underreams. The approach is largely analytical in nature and involves only a fraction of the computation required for a finite element analysis.

Consideration is given to the effects of anchor depth and inclination to the soil surface, and the spacing and number of underreams upon the elastic response of anchor systems. On the basis of the results from this study, a simple, approximate method for estimating the response of multiple underream anchors is proposed. This approach involves the use of several interaction charts, which are presented in the paper, and can be used as a hand method for estimating the load-displacement behaviour of quite general anchor systems to sufficient accuracy for most practical purposes. The use of the approximate approach is illustrated by an example.

1. INTRODUCTION

The increasing use of anchorage systems for both short and long term support of structures has indicated a need for a better theoretical understanding of anchor behaviour. In particular, although there has been a large increase in the number of experimental studies relating to anchor plate behaviour, until recently there have been relatively few theoretical studies. Previous elastic analyses of anchor plate behaviour have been largely restricted to the case of an isolated anchor plate (e.g. Fox,³ Douglas and Davis,² Selvadurai,⁸ Rowe and Booker⁶). Little consideration appears to have been given to the important practical case where the anchor system involves a number of underreams in series, as idealized in Figure 1.

In this paper a technique is developed for analysing the elastic behaviour of multiple underream anchor systems located at some depth beneath the surface of a homogeneous, isotropic half-space. The analysis involves subdivision of each underream into a number of sub-regions or elements and it is assumed that the force acting on a sub-region can be taken to be uniformly distributed over that sub-region. The theory of elasticity is then used to calculate an interaction matrix relating the element forces and element deflections.

Once the interaction matrix has been determined, it can be used to calculate the displacement and force distribution developed when an anchor is subjected to a prescribed resultant force or rigid body motion. Since only the underreams need to be divided into elements, the interaction matrix is relatively small, and consequently the approach only requires a fraction of the

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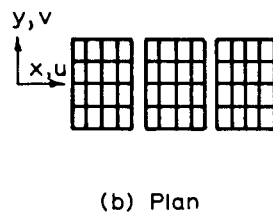
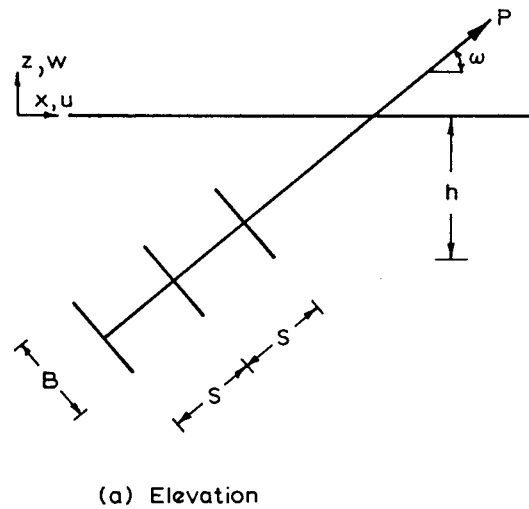


Figure 1. Typical problem configuration

computation involved in a finite element analysis and involves considerably less computation than conventional boundary integral approaches. A detailed investigation into the application of finite element techniques for the prediction of anchor behaviour up to and including collapse has been described by Rowe.⁵

In principle, the proposed method of analysis can be extended to incorporate the effects of finite layer depth and multilayer strata; however, the results obtained by Rowe and Booker⁶ for a single horizontally embedded circular anchor plate suggest that for practical purposes the techniques and solutions obtained for anchor systems in an elastic half-space will be sufficiently accurate for problems where the soil extends ten or more anchor diameters below the deepest underream. The displacements obtained assuming a half-space will be conservative for layers of finite depth.

Theoretical solutions for the elastic response of multiple underream anchors are presented, emphasis being placed upon the influence of anchor depth and inclination as well as the number and spacing of underreams.

An approximate hand method of estimating the elastic behaviour of multiple underream systems is then introduced and the use of the approach is illustrated for a number of cases.

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2. THEORY

Figure 1 shows a series of anchors A_1, \dots, A_n located in an elastic soil and connected as shown. Suppose that each anchor A_i is subdivided into m sub-regions or elements $D_{i1}, D_{i2}, \dots, D_{im}$ and that the force $\mathbf{F}_{ij} = (X_{ij}, Y_{ij}, Z_{ij})^T$ over each element D_{ij} can be assumed to be uniformly distributed. If only one element D_{kl} is loaded it follows from the theory of elasticity that the average deflection \bar{W}_{ij} of the element is given by a relation of the form

$$\bar{W}_{ij} = J_{ijkl} \mathbf{F}_{kl} \quad (1)$$

where the matrix J_{ijkl} has the form

$$J_{ijkl} = \begin{bmatrix} I_{xx} & I_{yx} & I_{zx} \\ I_{xy} & I_{yy} & I_{zy} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

and I_{xx}, I_{xy} , are influence factors. Typically I_{xy} represents the average y displacement over element D_{ij} due to the x component of a uniformly distributed load acting on element D_{kl} . A method for the determination of these influence coefficients for anchors embedded in a homogeneous elastic half-space has been given by Rowe and Booker.⁷

Now if all the elements D_{kl} , $l=1, \dots, m_k$ constituting the k th anchor plate A_k are considered to be loaded, then it follows from the principle of superposition that

$$\bar{W}_i = J_{ik} \mathbf{F}_k \quad (2)$$

where

$\mathbf{W}_i^T = (W_{i1}^T, W_{i2}^T, \dots)$ is the vector of average element displacements for the anchor plate A_i ,

$\mathbf{F}_k^T = (\mathbf{F}_{k1}^T, \mathbf{F}_{k2}^T, \dots)$ is the vector of element forces for the anchor plate A_k ,

and the influence matrix J_{ik} is given by

$$J_{ik} = \begin{bmatrix} J_{i1k1} & J_{i1k2} & \dots & J_{i1km} \\ J_{i2k1} & J_{i2k2} & \dots & J_{i2km} \\ J_{ink1} & J_{ink2} & \dots & J_{inkm} \end{bmatrix}$$

If attention is extended to all anchors, the principle of superposition shows that

$$\bar{\mathbf{W}} = J \mathbf{F} \quad (3)$$

where

$\mathbf{W}^T = (\mathbf{W}_1^T, \mathbf{W}_2^T, \dots, \mathbf{W}_n^T)$ is the vector of all average element deflections,

$\mathbf{F}^T = (\mathbf{F}_1^T, \mathbf{F}_2^T, \dots, \mathbf{F}_n^T)$ is the vector of all element forces,

and J is an $mn \times mn$ matrix defined by

$$J = \begin{bmatrix} J_{11} & J_{12} & \dots & J_{1n} \\ J_{21} & J_{22} & \dots & J_{2n} \\ \dots & \dots & \dots & \dots \\ J_{n1} & J_{n2} & \dots & J_{nn} \end{bmatrix}$$

The matrix J is easily shown to be symmetric and positive definite (e.g. see Connor¹).

If the anchors are rigid, then by definition there can be no deformation of the anchor itself and consequently any movement of the anchor plate from its original position will involve rigid body

displacement and rotation of the plate. The rigid movement of the plate may be resolved into three rigid displacement components (with respect to the x, y, z axes) and three rigid body rotation components. Thus the movement of anchor plate A_i may be represented in the form

$$\mathbf{W}_i = B_i \mathbf{R}_i \quad (4)$$

where

$$B_i = \begin{bmatrix} 1 & 0 & 0 & 0 & -\bar{z}_{i1} & y_{i1} \\ 0 & 1 & 0 & \bar{z}_{i1} & 0 & -\bar{x}_{i1} \\ 0 & 0 & 1 & -\bar{y}_{i1} & \bar{x}_{i1} & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & 0 & -\bar{z}_{im} & \bar{y}_{im} \\ 0 & 1 & 0 & \bar{z}_{im} & 0 & -\bar{x}_{im} \\ 0 & 0 & 1 & -\bar{y}_{im} & \bar{x}_{im} & 0 \end{bmatrix}$$

\mathbf{R}_i is the vector of six rigid body components and $\bar{x}_{ij}, \bar{y}_{ij}, \bar{z}_{ij}$ are the coordinates of the centroid of element D_{ij} . To this equation must be added the six equations of static equilibrium. If T_i denotes the tension in the shaft just above the anchor plate A_i and \mathbf{n}_i denotes a unit normal acting upward along this shaft, and if friction loss along the cable is neglected, then these equations may be written

$$B_i^T \mathbf{F}_i - T_i \begin{bmatrix} \mathbf{n}_i \\ \mathbf{m}_{ij} \end{bmatrix} + T_{i+1} \begin{bmatrix} \mathbf{n}_i \\ \mathbf{m}_{i+1i} \end{bmatrix} = 0 \quad (5)$$

where $\mathbf{m}_{ij} = \mathbf{n}_i \times \mathbf{r}_{aj}$ and \mathbf{r}_{aj} is the position vector of the point of attachment of shafts to the anchor A_j .

Finally there is the condition of constraint between adjacent anchors. It is found that

$$-\frac{T_{i+1}}{S_{i+1}} + (\mathbf{n}_{i+1}^T \mathbf{m}_{i+1i}^T) \mathbf{R}_i - (\mathbf{n}_{i+1}^T \mathbf{m}_{i+1i+1}^T) \mathbf{R}_{i+1} = 0 \quad (6)$$

where S_{i+1} is the stiffness of the $(i+1)$ th cable.

Equations (3)–(6) may now be combined to give a set having the form

$$\begin{bmatrix} J & -B & 0 \\ -B^T & 0 & C \\ 0 & C^T & \Phi \end{bmatrix} \begin{bmatrix} \mathbf{F} \\ \mathbf{R} \\ \mathbf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{K} \\ \mathbf{0} \end{bmatrix} \quad (7)$$

These $n(m+7) - 1$ equations may be solved for the mn element forces \mathbf{F} , the $6n$ rigid body components \mathbf{R} and the $n - 1$ unknown tensions \mathbf{T} .

3. TYPICAL SOLUTIONS

3.1 General details

The foregoing theory may be utilized to determine the elastic response of a multiple underream anchor at any inclination ω and for any number of underreams. The formulation is such that an anchor plate (underream) of general shape may be analysed by subdividing it into

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rectangular elements. The most practical case of a circular anchor may be approximated directly by appropriate subdivision into rectangular elements. However, it has previously been found (Rowe and Booker⁶), that to sufficient accuracy (i.e. to better than 1 per cent) the elastic response of a circular anchor is the same as that for a square anchor of equal area. This simplification leads to significant computational savings and consequently all the results presented in this paper have been determined for anchor plates which are square in section; the results are considered to be applicable to circular anchor plates of equal area.

The elastic response of a single rigid circular anchor at infinite depth has been obtained analytically by Selvadurai⁸ and may be expressed in the form

$$\delta_{\infty} = \frac{c_{\infty}}{BE} P \quad (8a)$$

where

δ_{∞} is the displacement of an anchor at infinite depth and with a diameter B subjected to an applied load P ,
 E is the Young's modulus of the soil, and

$$c_{\infty} \text{ (circle)} = \frac{(1 + \nu)(3 - 4\nu)}{8(1 - \nu)} \quad (8b)$$

where ν is Poissons ratio for the soil.

Equation (8a) may also be used for a square anchor provided the term c_{∞} is modified, viz:

$$c_{\infty} \text{ (square)} \approx \frac{\sqrt{(\pi)(1 + \nu)(3 - 4\nu)}}{16(1 - \nu)} \quad (8c)$$

Assuming that the anchor rod is relatively inextensible, the deflection of a multiple anchor system and/or an anchor system at finite depth may be given in terms of the result for a single anchor at infinite depth (equation 8) multiplied by a correction factor M_G , viz:

$$\delta = \delta_{\infty} M_G = \frac{c_{\infty}}{BE} M_G P \quad (9)$$

where M_G is a function of:

- anchor inclination angle ω (see Figure 1);
- the distance h between the soil surface and the bottom of the leading underream;
- the number of underreams;
- the spacing S between underreams;
- Poisson's ratio.

The correction factor M_G gives the displacement of a particular anchor system relative to the displacement of a single anchor plate which is at infinite depth and which is subjected to the same total load. For example, consider a two-underream anchor system at infinite depth. If the underreams are a great distance apart, then the displacement of this system will be half the displacement of a single anchor plate for the same total load P . This case corresponds to a value of $M_G = 0.5$. If the two underreams were not at a large spacing then interaction would increase the anchor system displacement relative to a single plate giving a value of M_G greater than 0.5.

It is considered that the most practical limiting case for multiple anchor systems is that involving rigid underreams. From Section 2, it will be appreciated that the elastic response of an anchor system will, to some extent, depend upon the number of subdivisions of the underreams.

However, it is found that to sufficient accuracy, the response of a rigid underream can be determined with relatively few subdivisions. In particular, a comparison of the analytical solution for a perfectly rigid underream with the results determined using this analysis indicates that 'rigid' underream solutions can be obtained to an accuracy of better than 6 per cent by the use of only one subdivision, and to an accuracy of better than 3 per cent by subdividing each underream into 16 elements (by symmetry, only 8 elements are actually used in the analyses). This accuracy would be adequate for most purposes, although clearly, more accurate results can be obtained by the use of additional subdivisions. More importantly, however, it is found that the correction factor M_G is relatively insensitive to the number of subdivisions.

Thus if

$$M_G(n) = \frac{\delta(n \text{ subdivisions})}{\delta_\infty(n \text{ subdivisions})}$$

so that

$$M_G = M_G(\infty) \quad \text{for a perfectly rigid underream}$$

then

$$M_G \approx M_G(n) \tag{10}$$

even for small values of n .

In fact, the correction factor M_G determined using results from a single subdivision of each underream ($n = 1$) was always within better than 2 per cent of the correct result for a rigid anchor and except for very shallow anchors ($h/B < 1.5$) was usually accurate to better than 1 per cent.

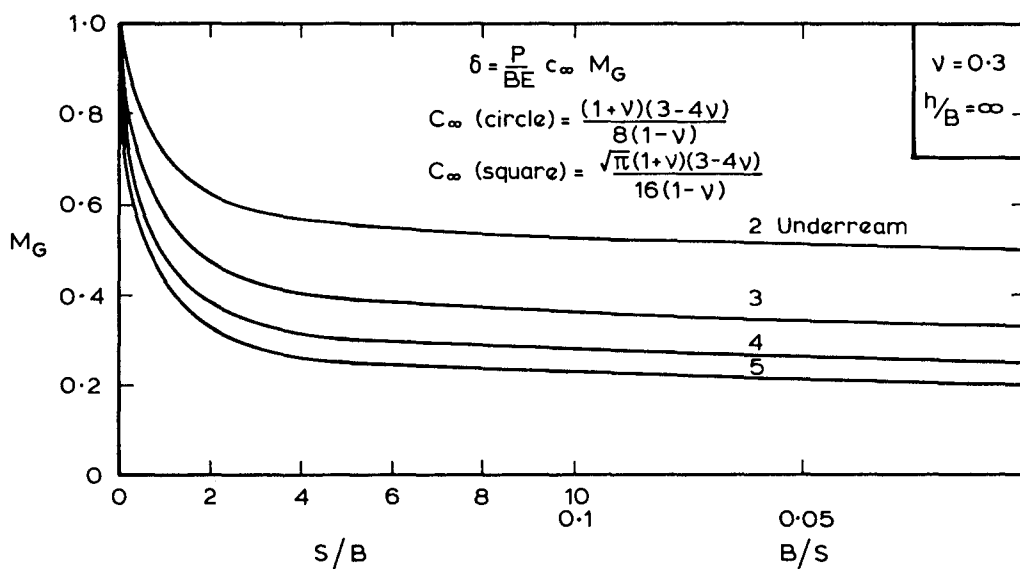
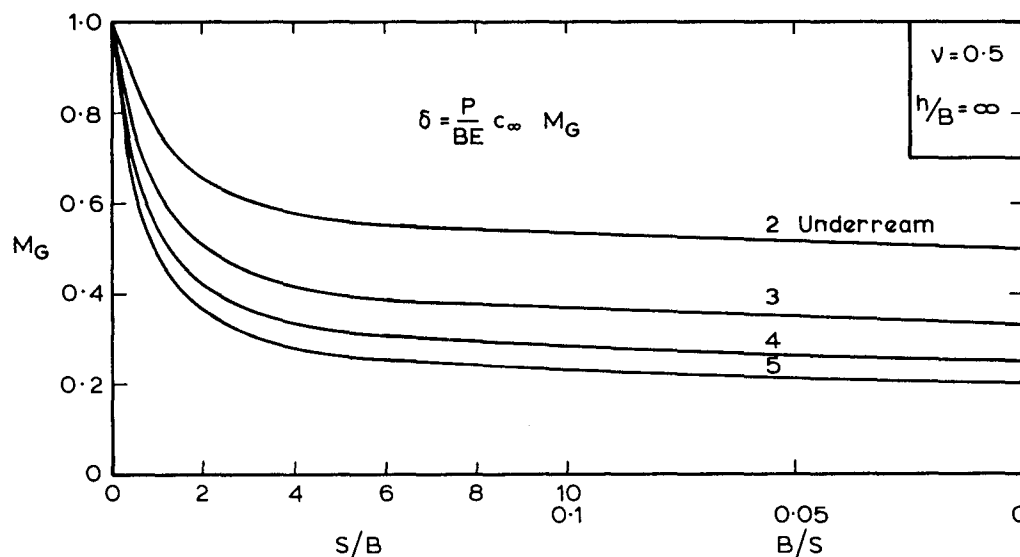
3.2 Specific results

To illustrate the relative importance of the various factors influencing the elastic response of multiple underream anchors, consideration was given to anchor systems consisting of between two and five underreams at a variable spacing S (see Figure 1). It was assumed that each underream could be idealized as a rough anchor plate which was fully bonded to the soil. This situation was considered to be the most practical limiting case for the application of elastic solutions since it has been shown by Rowe⁵ that separation of the anchor plate from the underlying soil is usually associated with significant plastic failure within the soil mass. The case of a rough, rigid anchor bonded to the elastic medium is one in which all displacements are prescribed at the anchor plate locations. This is the most straightforward case for numerical analysis, although the theory may be extended to deal with smooth anchors and breakaway along the lines indicated by Rowe and Booker.⁶

The correction factor M_G for multiple anchors at infinite depth is given in Figures 2(a) and 2(b) as a function of anchor spacing for soils with Poisson's ratios $\nu = 0.3$ and $\nu = 0.5$ respectively. For a system with two underreams, the presence of the second underream significantly reduces the displacement of the anchor system (compared with a single anchor) leading to more than a 40 per cent reduction in the displacement of the system for anchor spacings greater than three anchor widths. The theoretical results indicate significant interaction effects for anchors at spacings of less than two anchor widths and in practice one might anticipate even greater interaction owing to disturbance effects upon the soil during the underreaming process.

As one would expect, increasing the number of underreams (for a constant spacing between underreams) decreases the displacement of the anchor system for the same total applied load,

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Figure 2(a). Displacement reduction factor M_G for an anchor system at infinite depth: $\nu = 0.3$ Figure 2(b). Displacement reduction factor M_G for anchor system at infinite depth: $\nu = 0.5$

and indeed for most practical situations (i.e. providing the dimensionless anchor spacing S/B is not significantly less than two) it is more beneficial to introduce an additional underream than to increase the spacing between underreams.

The efficiency of a deep anchor system depends upon the number and spacing of underreams as well as Poisson's ratio for the soil. As mentioned, increasing the number of underreams generally improves the performance of the anchor system; however, there is significant

interaction between underreams. The efficiency of a system may be assessed by comparing the displacement of that system for a given load to the displacement that would be obtained if there were no interaction (i.e. as the dimensionless spacing S/B tends to infinity the stiffness of the anchor system is simply the sum of the component stiffness of the individual underreams). The percentage increase in displacement due to interaction between underreams is given in Table I for deep anchor systems with two and five underreams.

Table I. Percentage increase in anchor displacement due to interaction
(a) 2 underreams

Poisson's ratio	Dimensionless spacing S/B			
	2	3	5	10
0.3	25	17	11	5
0.5	31	22	13	7

(b) 5 underreams				
Poisson's ratio	Dimensionless spacing S/B			
	2	3	5	10
0.3	65	45	28	14
0.5	80	55	33	16

For a two-underream system, notable interaction occurs for anchor spacings S/B less than or equal to 5. Increasing the spacing above 5 would generally lead to less than a 10 per cent increase in displacement, and a spacing of 5 may be considered to be a practical spacing at which relatively little interaction occurs for the two-underream systems. The interaction effects increase appreciably with the number of underreams, and for the five-underream case interaction will increase the displacement by in excess of 65 per cent for $S/B = 2$, and even at relatively large spacing ($S/B = 10$) there is more than a 15 per cent increase in displacement owing to interaction. Notice also that for relatively close spacings ($S/B < 5$) the interaction is dependent upon Poisson's ratio and that the incompressible case ($\nu = 0.5$) gives rise to the greatest interaction.

The presentation of results for an anchor at infinite depth (Figure 2) raises the practical question as to what constitutes 'infinite depth', since it may be anticipated that the presence of the stress free surface will tend to increase the displacement of an anchor system. Figure 3 shows the variation of the correction factor M_G with the embedment ratio of the leading underream (i.e. embedded depth h to the top underream divided by underream width B) for two anchor systems with a vertical axis (inclination angle $\omega = 90^\circ$). For closely spaced underreams the proximity of the soil surface may increase the anchor displacement by more than 40 per cent compared with an anchor system at infinite depth. The influence of the free surface upon the anchor system decreases with the anchor spacing, but even at a relatively large spacings of five anchor widths, the use of the 'infinitely deep' anchor solution would underestimate the displacement of the system appreciably for embedment ratios less than 5. It will be recognised that increasing either the number of underreams or the spacing between underreams will lead to an increase in the distance between the bottom of the anchor system and the soil surface. This effectively increases the overall anchor depth even though the position of the top underream is

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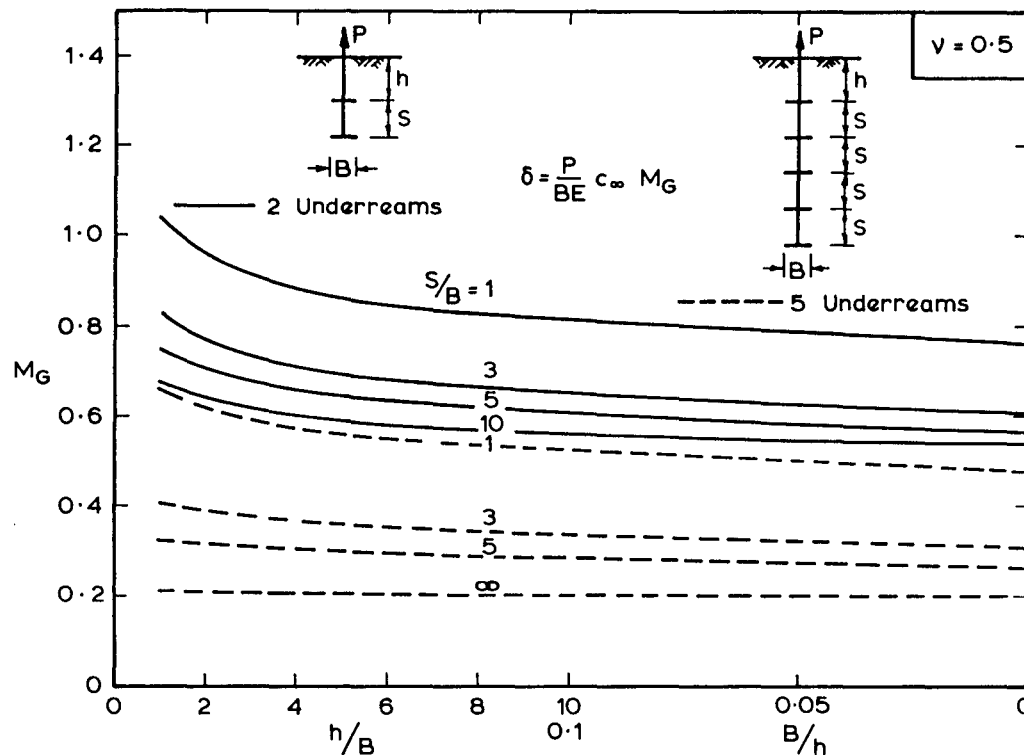


Figure 3. Effect of embedment upon anchor performance: $\omega = 90^\circ$, $\nu = 0.5$

constant for a given h/B . Consequently, the influence of the free surface is a function of both the number and spacing of underreams, and so the critical embedment ratio (h/B), at which the elastic response of the anchor would be the same as if it were 'infinitely deep', will also be a function of these parameters. In spite of this complexity of anchor system behaviour, it would appear that for practical purposes multiple underream anchors with a vertical axis ($\omega = 90^\circ$) may generally be considered to be 'deep' for embedment ratios ranging from 5 to 10, and the elastic response of these 'deep' anchors may be predicted to an accuracy of better than 10 per cent using solutions which totally neglect the presence of the soil surface (e.g. Figure 2).

The displacement of multiple underream anchors with a vertical axis (Figure 3) may be compared with the displacement of an incompressible vertical pile with an equivalent length to diameter ratio. As might be expected, the displacement of the multiple anchor system is greater than the displacement given by the Poulos and Davis⁴ solutions for an equivalent incompressible pile; however, the difference between the two types of behaviour decreases with decreasing underream spacing. Thus the displacements of multiple anchors with both a spacing ratio (S/B) and embedment ratio (h/B) equal to one (as given in Figure 3) are approximately 12 per cent greater than the displacement of a similar incompressible pile. This difference is reduced to about 8 per cent for a spacing ratio of one and embedment ratio of zero. Because of the rough rigid nature of the anchor plates, decreasing the anchor spacing below one largely immobilizes the soil between the plates, and the displacement of the anchor system tends toward that of a geometrically similar incompressible pile. Despite this theoretically similar response between an anchor system with closely spaced underreams and an incompressible pile, it should be

recognised that in practice the different disturbance effects associated with the installation of the two foundation systems would be expected to result in some difference in their 'elastic' response.

Anchor systems, with a horizontal axis ($\omega = 0^\circ$) and with close to moderately spaced underreams tend to be less sensitive to the proximity of the free surface than anchor systems with vertical axes. This trend is reversed for shallow underreams at large spacings because all the underreams are at the same depth irrespective of spacing. Anchor systems with a horizontal axis and a single underream may be considered to be 'deep' at an embedment ratio of 2.5; however, with from two to five underreams this depth increases to between four and seven underream widths, depending upon the number and spacing of the underreams. From the foregoing, and from a comparison of Figures 3 and 4, it can be seen that for shallow anchors the performance of

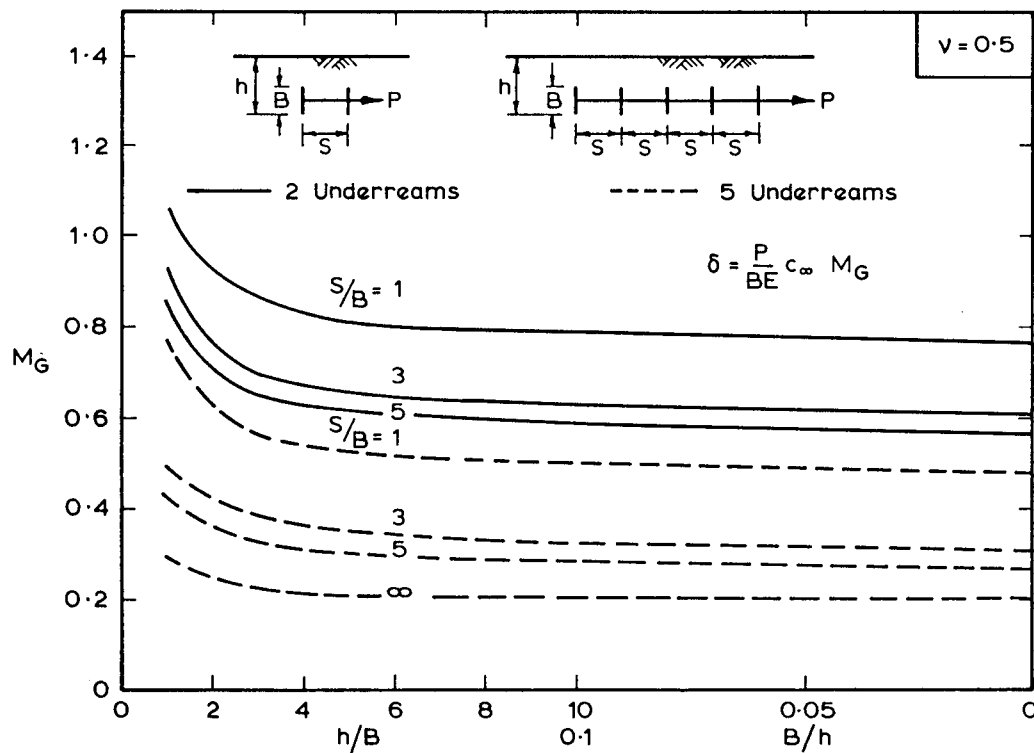


Figure 4. Effect of embedment upon anchor performance: $\omega = 0^\circ$, $\nu = 0.5$

the anchor system will be dependent upon the inclination of the anchor system to the free surface. Clearly for deep anchors, the response of the anchor system will be independent of the inclination angle ω .

The variation of the anchor system displacement with inclination angle ω is shown in Figure 5 for an anchor system with two underreams resting in an incompressible soil. The embedment ratio h/B represents the dimensionless distance to the base of the leading underream. In comparing the results for different inclinations, it should be remembered that the case where $\omega = 0^\circ$ is a special case since the embedded depth of both underreams is independent of the

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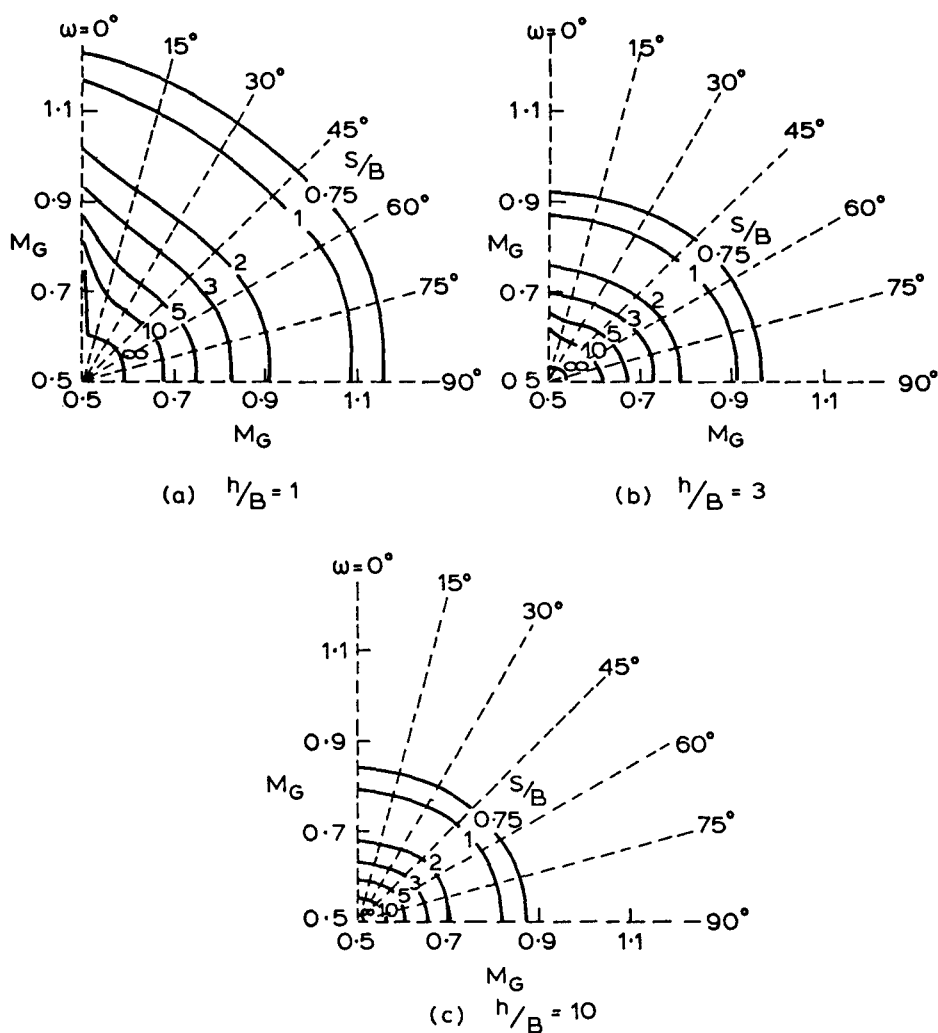


Figure 5. Effect of anchor inclination upon response of a two-underream system: $\nu = 0.5$

anchor spacing; i. all other cases the depth to the second underream increases with increasing spacing S/B .

The results give, in Figure 5 indicate that, except for the special case as the inclination angle approaches 0° , the displacement of the anchors was relatively insensitive to inclination angle. In fact, the displacement of anchor systems with inclination angles greater than or equal to 15° could be predicted to an accuracy of better than 7 per cent using solutions for $\omega = 90^\circ$. For embedment ratios greater than 3, this assertion is true for all values of inclination in the range from 0° to 90° .

Similar calculations for anchor systems involving two, three and five underreams in soils with Poisson's ratio equal to both 0.3 and 0.5 showed that the aforementioned case with $\nu = 0.5$ and two underreams is the most critical multiple anchor case when considering the effects of anchor inclination, and consequently the above conclusions regarding the general insensitivity of

