A method of estimating surface settlement above tunnels constructed in soft ground

R. K. Rowe and K. Y. Lo
Faculty of Engineering Science, University of Western Ontario, London, Ont., Canada N6A 5B9
AND
G. J. Kack
Thurber Consultants Ltd., Edmonton, Alta., Canada T6E 1X7

Received December 18, 1981
Accepted July 21, 1982

A technique suitable for the analysis of lined tunnels constructed in soft soil is described. This technique permits consideration of the soil-lining interaction, lining weight, and plastic failure within the soil. Particular attention is given to the simulation of loss of ground including the annular void created by the difference between the tunnelling machine and lining diameters. The analysis is used in a parametric study to identify potentially significant factors affecting the prediction of surface settlement due to tunnel construction. Consideration is given to the effects of elastic modulus; the distance between the tunnel and the underlying rigid base stratum; plastic failure; initial stresses; ground loss (including the annular void); and the injection of clay grout into the tailpiece void. The results of this analysis indicate that elastic anisotropy, ground loss, and clay grout injection are all important factors that are not usually considered in analyses. In particular, a parameter called the gap is defined in the paper and is used in an attempt to incorporate practical tunnelling experience into the calculation of surface settlement. Furthermore, it is shown that in predicting surface settlements underestimation of the true elastic modulus below the tunnel may decrease the predicted surface settlement and lead to overestimation of the invert heave. Underestimation of the modulus above the tunnel may also decrease the predicted surface settlement for situations where there is limited ground loss and extensive plastic failure.

L'article décrit une méthode d'analyse applicable aux tunnels avec revêtement construits dans les sols mous. Cette technique permet de prendre en compte l'interaction sol–revêtement, le poids du revêtement, et la rupture plastique dans le sol. On porte une attention particulière à la simulation de la surexcavation, y compris le vide annulaire crée par la différence de diamètre entre les diamètres du tunnelier et du revêtement. L'analyse est utilisée dans une étude paramétrique pour identifier les facteurs importants affectant la prédiction du tassement de surface du à la construction du tunnel. On considère les effets du module d'élasticité, de la distance entre le tunnel et le substratum rigide, de la rupture plastique, des contraintes initiales, de la surexcavation (incluant le vide annulaire), et de l'injection d'un coulis d'argile dans le vide à l'arrière du tunnelier. Les résultats de cette analyse indiquant que l'anisotropie d'élasticité, la surexcavation et l'injection de coulis sont toutes des facteurs importants qui ne sont normalement pas pris en compte dans les analyses. En particulier un paramètre appelé "gap" est défini dans l'article et est utilisé dans une tentative d'incorporation de l'expérience de la pratique des tunnels dans le calcul des tassements de surface. De plus on montre que, lors de la prédiction des tassements de surface, une sous estimation du module d'élasticité au-dessous du tunnel peut réduire la prédiction du tassement de surface et sur-estimer le soulèvement du plancher du tunnel. Une sous estimation du module au dessus du tunnel peut aussi réduire le tassement de surface calculé dans les situations où la surexcavation est limitée et où la rupture plastique est très développée.

[Traduit par la revue]

Can. Geotech. J., **20**, 11–22 (1983)

Introduction

Rapid growth in urban development has resulted in increased demand for the construction of water supply, sewage disposal, and transportation systems. Tunnels are an essential component of these schemes and constitute a major portion of project expenditure.

Recent advances in tunnelling technology reduce construction time with consequent decreases in cost. However, even with modern equipment, experience has shown (e.g., Peck 1969; Schmidt 1969; Attewell and Farmer 1974) that subsidence invariably occurs in areas above and adjacent to tunnels passing through soft soil deposits. These deformations may significantly affect nearby structures and need to be considered during design. Unfortunately, theoretical advances have not kept pace with the recent advances in tunnelling technology and at present there is no generally valid method of predicting ground subsidence before construction.

Although the finite element method is well recognised as a potential tool for predicting deformations due to tunnelling, its application to the problem of predicting subsidence in soft soil is extremely limited. This situation arises, in part, from uncertainties regarding the applicability of idealized analyses to real conditions. To allow confident use of finite element techniques in tunnel design, it is necessary to identify the effects of different problem idealizations upon predicted soil movements and to relate this to the observed behaviour from well-documented case histories.

This paper represents the first part of such a study and

0008-3674/83/010011-12$01.00/0

has two objectives. The first is to describe a technique suitable for estimating the deformations caused by the installation of a lined tunnel in soft ground. The second is to use this technique in a parametric study that will identify potentially significant factors affecting the determination of surface settlement caused by tunnelling.

The method of analysis to be described is capable of considering soil–lining interaction where the lining may be assumed to be rigid or where the actual lining stiffness is directly considered. This technique is considered most appropriate for tunnels constructed using a tunnelling machine. The method of analysis and the insight afforded by the parametric study will be used in the analysis of six case histories in two subsequent publications (Rowe and Kack 1983; Lo and Rowe 1982).

Principal assumptions—soil model and finite element mesh

The analyses reported in this paper were performed using the authors' plane strain elasto-plastic finite element program (EPTUN). The relative advantages and disadvantages of plane strain analyses, as opposed to three-dimensional analysis, have been discussed in the literature (e.g., Ghaboussi *et al.* 1978) and it is generally concluded that the plane strain solutions do provide useful information. The true three-dimensional analyses that have been performed also provide some useful insight, although they have generally been based on the assumption that the soil is elastic. This is not considered a good idealization of soft soil. Clearly, it would be desirable to perform a true three-dimensional elasto-plastic analysis; however, the costs of data preparation and computation involved in such an analysis are so high as to make the analysis impractical for design purposes. The approach described in this paper is considered to be sufficiently flexible and economic (both in terms of man-hours and computer-hours) to find application in practice. It should also be noted that some of the less tangible factors (such as workmanship) affecting the prediction of surface settlement are equally uncertain with two- or three-dimensional analysis.

This paper is concerned with the prediction of the final equilibrium settlements and all the parameters used are drained parameters. Thus the settlements include both the immediate and the consolidation settlements. Elastically, the soil was assumed to be either isotropic or cross-anisotropic with a rotational symmetry about the vertical axis. The plastic behaviour was modelled using a Mohr–Coulomb failure criterion and a nonassociated flow rule of the form proposed by Davis (1968). It was assumed that the drained plastic response of the soil would be isotropic and that, for soft or loose soil, there would be no dilatancy. Prior to tunnel excavation, the major and minor principal stresses were assumed to be in the vertical and horizontal directions respectively. The coefficient of earth pressure at rest (K_0) was taken to be less than unity. For the purposes of the parametric study, the tunnel lining was assumed to be rigid. (The effect of lining flexibility was considered by Lo and Rowe (1982).)

In the following sections, consideration will be given to the importance of elastic modulus variations; the distance below the tunnel to a relatively rigid stratum; elastic anisotropy; the plasticity parameters; K_0; the initial "gap" between the lining and the soil (to be defined in the next section); and the injection of clay grout into the tailpiece void.

The elastic and elasto-plastic analyses performed with EPTUN used eight-noded isoparametric elements. The finite element mesh was selected after consideration of the results from a number of analyses performed for different mesh arrangements and for lateral boundaries at different positions. Details regarding the finite element procedure have been given by Kack (1981).

Construction simulation

Construction of the tunnel was simulated by deducing the tractions that would be acting around the surface of the tunnel (prior to excavation) and then removing these tractions. This approach is valid for all values of K_0 (e.g., see Kulhawy 1974) and is considered to provide a reasonable approximation of the change in stress that occurs with the advance of the tunnelling machine. Construction with compressed air may be simulated by reducing (or increasing) tractions until they correspond to the air pressure.

In simulating tunnel construction, consideration must be given to two key factors influencing possible deformation near the tunnel lining. Firstly, local stresses induced by advancing the tunnelling machine will create a zone of remoulded soil around the tunnel. The extent of this zone will depend, in part, upon workmanship and construction factors such as shield performance and rate of advance. Consolidation of this remoulded zone will result in a volume decrease of the soil, thereby providing additional freedom for settlement of the overlying soil.

Secondly, with the advance of the tunnelling machine, soil in front of the heading will move both radially and axially towards the face (e.g., see Peck 1969). Thus the soil that forms the final cut surface of the tunnel will have originally been located at some greater distance from the centreline of the tunnel. The volume between the final cut surface and the original (undisturbed) position of this soil represents a loss of ground. Additional loss of ground is represented by the annular void which is the difference between the mined diameter and the external diameter of the lining.

The net effect of these factors may be approximately incorporated in a plane strain analysis in terms of a void that incorporates both the loss of ground and the volume change in the remoulded soil. The magnitude of this void may be expressed in terms of gap parameter. If the invert of the tunnel rests on the underlying soil, then the gap is (allowing for heave) the vertical distance between the crown of the tunnel and the original position (i.e., prior to tunnelling) of the soil which is now directly above the crown.

Since the gap is finite, an unlined tunnel analysis is strictly only valid provided the soil does not come into contact with the lining. In practice, contact will occur and the weight of the tunnel will be transferred to the underlying soil while the lining will limit the possible deformations of the soil. In this analysis, the deformation of the cut tunnel surface was monitored during unloading. Once the cut surface comes into contact with the tunnel lining, the interaction between the soil and the lining may be analysed using the soil–structure interaction theory proposed by Rowe *et al.* (1978). The application of this theory to a flexible tunnel lining will be briefly described in the following section.

Implementation of soil–structure interaction theory

Consider a tunnel lining embedded in an elasto-plastic soil mass. The tractions acting upon the soil and lining may be considered to take the form of nodal forces. Within any load increment these forces will consist of known forces due to applied tractions acting directly on the body, and unknown nodal forces $\dot{F}_u$ acting between the soil and lining.

Supposing for the moment that incremental nodal forces acting between soil and lining are also known; then the initial stress approach (Zienkiewicz *et al.* 1969) can be used to analyse the soil, and the vector of nodal displacements at the soil–tunnel interface $\dot{\rho}$ may be written in the form

$$[1] \qquad \dot{\rho}^{(n)} = \dot{\rho}_K{}^{(n)} + I_E \dot{F}_u{}^{(n)}$$

where $\dot{\rho}_K$ is the vector of incremental nodal displacements at the soil–tunnel interface due to the known incremental nodal forces acting on the soil (including residual forces arising from the initial stress formulation); $\dot{F}_u$ is the vector of incremental nodal forces due to the lining acting on the soil and is initially unknown; and I_E is the matrix of influence coefficients for the soil which can be determined initially and will remain constant for each iteration and load step. Each element I_{ij} of this matrix corresponds to the displacement at the ith node due to a unit load applied at the jth node, where both the ith and jth nodes lie on the excavated surface. The superscript n denotes the nth estimate of that variable.

If the tunnel lining is considered as a separate body, subjected to a self-weight W and the (unknown) nodal forces F_u, it will be free to undergo incremental rigid body motions $\dot{\theta}$. Thus, the nodal displacements $\dot{\rho}_e$ at the lining-soil interface may be written in the form

$$[2] \qquad \dot{\rho}_e{}^{(n)} = \dot{\rho}_{Ke}{}^{(n)} - I_e \dot{F}_u{}^{(n)} + A\dot{\theta}^{(n)}$$

where $\dot{\rho}_{Ke}$ is the vector of incremental nodal displacement of the lining, relative to the invert, due to known incremental forces acting on the lining (e.g., self-weight of the lining) and I_e is the matrix of influence coefficients for the lining. This matrix is analogous to the matrix I_E for the soil. Thus the ijth component of matrix I_e is the displacement of the ith lining node due to a unit force on the jth node; A is a matrix relating interface deflections to the rigid body motions.

If the vertical plane through the tunnel is also an axis of symmetry, then there will be only one rigid body motion (i.e., vertical displacement) and the matrix A reduces to a vector that has a unit value for vertical components of displacement and zero for horizontal components of displacement.

Displacement compatibility at all nodes in contact along the interface between the soil and lining may be invoked by equating $\dot{\rho}^{(n)}$ and $\dot{\rho}_e{}^{(n)}$ as defined by [1] and [2] respectively. This gives:

$$[3] \qquad (I_E + I_e)\, \dot{F}_u{}^{(n)} - A\dot{\theta}^{(n)} = \dot{\rho}_{Ke}{}^{(n)} - \dot{\rho}_K{}^{(n)}$$

If there are m unknown nodal forces and p rigid body motions, then [3] represents m equations in $(m + p)$ unknowns. The remaining equations may be derived from the observation that the lining must be in a state of equilibrium. Thus the net vertical force acting on the lining must be equal to the tunnel weight and there will be no net horizontal force or moment. These equations of equilibrium may be written in the form

$$[4] \qquad A^T(\dot{F}^{(n)} - \dot{W}) = 0$$

where $A^T\dot{W}$ is the increment in weight of the tunnel.

Equations [3] and [4] may be written as the simple interaction equation

$$[5] \qquad \begin{bmatrix} (I_E + I_e) & -A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} \dot{F}_u{}^{(n)} \\ \dot{\theta}^{(n)} \end{bmatrix} = \begin{bmatrix} \dot{\rho}_{Ke}{}^{(n)} - \dot{\rho}_K{}^{(n)} \\ -A^T\dot{W} \end{bmatrix}$$

Equation [5] may be solved to determine $\dot{F}_u{}^{(n)}$ and $\dot{\theta}^{(n)}$. Once these quantities are known the displacements within the soil mass may be directly determined. The process is repeated until convergence is achieved. It is noted that the number of nodes in contact with the lining will generally increase during the loading and the number of unknown forces $\dot{F}_u$ will increase as the distance between the soil and lining decreases. For simplicity, the formulation indicated in [5] is for a perfectly rough lining; however, problems in which slip occurs at the interface can also be solved using this method (e.g., see Rowe *et al.* 1978).

If the tunnel lining is assumed to be rigid, then [5] reduces to

$$[6] \quad \begin{bmatrix} I_{\mathrm{E}} & -A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} \dot{F}_{\mathrm{u}}^{(n)} \\ \dot{\theta}^{(n)} \end{bmatrix} = \begin{bmatrix} -\dot{\rho}_{\mathrm{K}}^{(n)} \\ -A^T \dot{W} \end{bmatrix}$$

This approach derives its power from the fact that it is only necessary to triangularise the elastic stiffness matrix once for all iterations and load steps. Furthermore, consideration may be given to a wide range of interface conditions (including gap closure and slip) without the introduction of special joint or contact elements and without reforming the stiffness matrix.

Application of soil–structure interaction analysis to the prediction of subsidence: a parametric study

Several investigators have recognized that unlined tunnel analyses are not appropriate for predicting settlements due to the construction of lined tunnels in soft ground (e.g., see Ghaboussi *et al.* 1978; Tan and Clough 1980). However, no general investigation of the interaction between soil properties, lining gap, and grout pressure appears to have been attempted for lined tunnels.

The presence of the lining will influence the soil deformations in two respects. Firstly, the displacement of the soil will be affected by the dead load of the lining and the average live load within the tunnel. These loads are not insignificant and a reasonable estimate of the dead and live loads may represent up to 50% of the weight of soil removed during excavation.

Secondly, the physical presence of the lining will limit deformations of the surrounding soil and may give rise to a stress redistribution within the soil. In this study it is assumed that the lining rests on the soil directly beneath the invert of the tunnel. No other assumption is made regarding the final position of the lining (which may move with the underlying soil) since the final position of soil and lining may be deduced directly using the analysis of the previous section. To identify the effect of the gap parameter upon soil movements, consideration will be given to excavations corresponding to gaps of 90, 115, 130, and 160 mm.

For definiteness, the tunnel geometry was selected to correspond to the Thunder Bay sewer tunnel (Palmer and Belshaw 1980) which had an external lining diameter of 2.38 m and a centreline at a depth of 10.5 m. Unless otherwise stated, in the following analyses it will be assumed that: the soil has a unit weight of $14.7\,\mathrm{kN/m^3}$, $K_O = 0.6$, and anisotropic elastic ratios (case V) given in Table 1; the gap is 115 mm; the lining is rigid; grout pressure is zero; and the tunnel has a dead plus live load weight of 30 kN per metre length.

Six different hypothetical variations in modulus with depth will be considered as shown in Fig. 1. The first three profiles represent unrealistically low modulus values and are specifically chosen to illustrate the effect of a low modulus. The remaining three profiles (Fig. 1*d–f*) are considered to be more representative of the elastic modulus profiles that might be encountered in soft–firm soil deposits.

In the following study, the "surface" settlement profile will correspond to settlements 2 m below the soil surface. This depth was selected to correspond to the depth at which shallow foundations or services that might be affected by tunnelling are likely to be located. The general conclusion regarding the effect of different parameters on this "surface settlement" are equally valid at the actual surface of the deposit.

Effect of nonhomogeneity and the distance from the tunnel to a relatively rigid base stratum

Excavation of a tunnel gives rise to a local decrease in radial stress and increase in tangential stress around the tunnel. Furthermore, from a consideration of the vertical equilibrium, it is obvious that there will also be a general net decrease in the vertical stress due to removal of the soil and that this must be balanced by a decrease in vertical reaction at the base of the soil deposit. This will induce strains within the soil mass below the tunnel. These strains result in an upward movement of the soil that is superimposed on the settlements due to the local effects of stress removal.

This upward displacement for deep soil deposits (i.e., deposits extending more than ten tunnel diameters below the tunnel) is directly proportional to the distance between the tunnel and the underlying stiff base stratum. Thus it may be inferred that the upward movement will be infinite for a homogeneous layer of infinite depth.

The increase in upward vertical movement with increasing depth to the underlying stiff stratum corresponds to a rigid body motion that reduces the magnitude (but does not affect the distribution) of the surface settlement. Clearly, the magnitude of this rigid body motion will depend on the modulus of the underlying soil. Thus, for deep deposits, theoretical analyses may give rise to completely erroneous displacements if the modulus of the soil beneath the tunnel is significantly underestimated. This will be demonstrated below.

Elastic analysis

The settlement profile at a depth of 2 m below the surface of an isotropic elastic soil is shown in Fig. 2 for a number of assumed variations in elastic modulus with depth. Curve (*a*) in Fig. 2 corresponds to a homogeneous deposit with $E' = 1.5\,\mathrm{MPa}$. Although this modulus is already lower than could reasonably be expected, the centreline settlement is still very small. This is partly

TABLE 1. Anisotropic ratios

Case No.	0–7.9 m depth				7.9–12.9 m depth				12.9–24.5 m depth			
	E_h/E_v	G_{vh}/E_v	ν_h	ν_{vh}	E_h/E_v	G_{vh}/E_v	ν_h	ν_{vh}	E_h/E_v	G_{vh}/E_v	ν_h	ν_{vh}
I	1.	0.4	0.25	0.25	1.	0.4	0.25	0.25	1.	0.4	0.25	0.25
II	1.	0.4	0.2	0.2	1.	0.4	0.2	0.2	1.	0.4	0.2	0.2
III	1.	0.4	0.0	0.0	1.	0.4	0.0	0.0	1.	0.4	0.0	0.0
IV	1.	0.25	0.2	0.2	From 1.	0.3	0.2	0.2 at 7.9 m	1.	0.4	0.2	0.2
					to 1.	0.4	0.2	0.2 at 12.9 m				
V	0.5	0.25	0.2	0.2	From 0.5	0.3	0.2	0.2 at 7.9 m	1.	0.4	0.2	0.2
					to 0.7	0.4	0.2	0.2 at 12.9 m				
VI	0.5	0.25	0.2	0.2	From 0.5	0.3	0.2	0.2 at 7.9 m	0.7	0.4	0.2	0.2
					to 0.5	0.4	0.2	0.2 at 12.9 m				
VII	0.5	0.25	0.2	0.2	0.5	0.25	0.2	0.2	1.	0.4	0.2	0.2
VIII	0.5	0.25	0.0	0.0	0.5	0.25	0.0	0.0	1.	0.4	0.0	0.0

NOTES: E_v = vertical Young's modulus; E_h = horizontal Young's modulus; G_{hv} = independent shear modulus; ν_h = Poisson's ratio for the effect of horizontal stress on complementary horizontal strain; ν_{vh} = Poisson's ratio for the effect of vertical stress on horizontal strain.

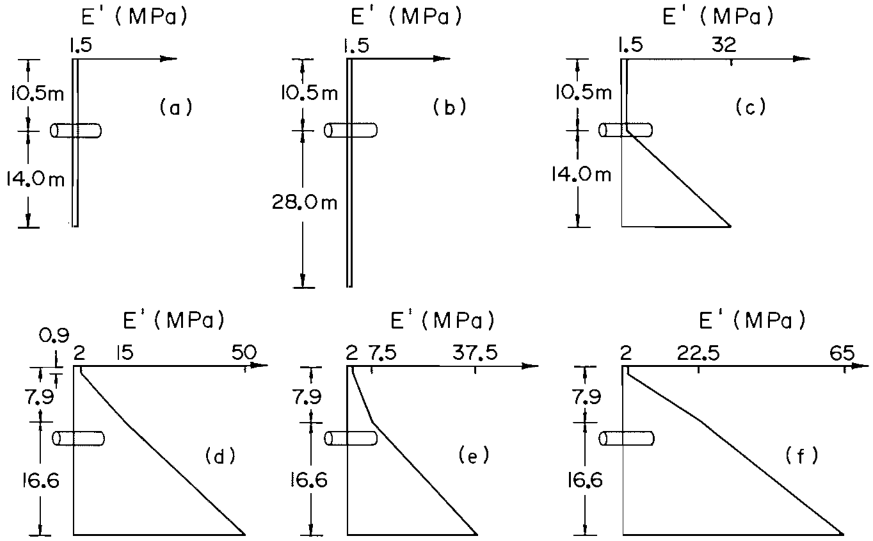

FIG. 1. Some assumed variations in elastic modulus with depth.

because a heave of more than 70 mm occurs at the invert of the tunnel. This heave severely limits the settlement at the crown for finite initial gap. Curve (*b*) indicates that the heave will increase with increasing depth of soil beneath the tunnel and in this case there is a general heave of the soil mass. A comparison of the settlement indicated for curves (*a*) and (*c*) further illustrates the effect of heave. In case (*c*), the soil modulus increases linearly with depth below the centre of the tunnel thereby reducing the invert heave to 16 mm, and increasing the maximum settlement (compared with curve (*a*)) by more than a factor of four.

The foregoing results imply that the surface settlement will increase with decreasing distance from the tunnel to the base stratum and increasing stiffness of the soil below the tunnel. Thus, in predicting surface settlement due to tunnelling, it is not conservative to assume that the soil below the tunnel has a low elastic modulus. (The fact that relatively small heaves are observed for most tunnels in soft soil is indirect evidence that the extension modulus of these soft soils is quite high.)

The effect of soil–lining interaction upon the settlement is apparent from a comparison of the settlement troughs for case (*c*) with both a lined and unlined tunnel. In this case, the unlined analysis gives a more realistic estimate of the settlement one might expect above a tunnel in soft soil but for quite the wrong reasons.

None of the soil profiles shown in Fig. 1*a*–*c* are considered realistic representations of the actual mod-

TABLE 2. Plasticity parameters and K_o

	Peat with some silt and sand, 0–0.9 m depth			Loose sand–silty sand, 0.9–7.9 m depth			Soft to firm silty clay–clay, 7.9–12.9 m depth			Firm to stiff silty clay–clay, 12.9–24.5 m depth		
Case No.	c' (kPa)	ϕ' (°)	K_o	c' (kPa)	ϕ' (°)	K_o	c' (kPa)	ϕ' (°)	K_o	c' (kPa)	ϕ' (°)	K_o
1	5	22	0.6	0	30	0.5	0	22	0.6	10	25	0.6
2	5	22	0.6	0	30	0.5	0	22	0.5	10	25	0.5
3	5	22	0.6	0	30	0.5	0	22	0.7	10	25	0.7
4	5	22	0.6	0	27	0.55	0	22	0.6	10	25	0.6
5	5	22	0.6	0	33	0.45	0	22	0.6	10	25	0.6
6	5	22	0.6	0	30	0.5	15	22	0.6	10	25	0.6
7	5	22	0.6	0	30	0.5	0	18	0.7	10	25	0.7
8	5	22	0.6	0	30	0.5	0	26	0.55	10	25	0.7

ulus profile in soft deposits and the results of these analyses serve to illustrate that, in the analysis of lined tunnels, the modulus distribution below the tunnel and the depth to the base stratum may have a significant effect on the prediction of surface settlement. This conclusion is further emphasized by considering the settlement trough (see Fig. 2*d*) obtained for what is considered to be a more realistic variation in modulus for a soft soil (see Fig. 1*d*). Here the heave of the tunnel invert has been reduced to 10 mm and, since the soil does not come into contact with the crown of the tunnel, it would appear that the small settlements are the result of the small strains associated with the high soil modulus above the tunnel rather than the heave.

Both the magnitude and shape of the settlement trough are of practical significance. The normalized settlements shown in Fig. 2 indicate a much wider settlement trough than is normally encountered in practice. The poor prediction of the shape of settlement trough that is obtained from an isotropic elastic analysis arises from the inadequacy of the problem idealization and is not simply due to uncertainty regarding the actual distribution of soil modulus.

Elasto-plastic analysis

In modelling the behaviour of lined tunnels in soft soil, it is usually necessary to consider plastic failure within the soil. Adopting, initially, a soil profile with the plasticity parameters (case 1) given in Table 2, elasto-plastic analyses were performed for the elastic modulus profile in Fig. 1*d* and two additional modulus profiles shown in Fig. 1*e* and *f*. The settlement troughs obtained for these three modulus profiles are given in Fig. 3. With a gap of 115 mm, there is relatively little variation in the maximum settlement obtained for all three cases despite an almost threefold variation in elastic modulus near the tunnel. These results demonstrate the dominant effect of contained plastic failure in determing the settlement trough.

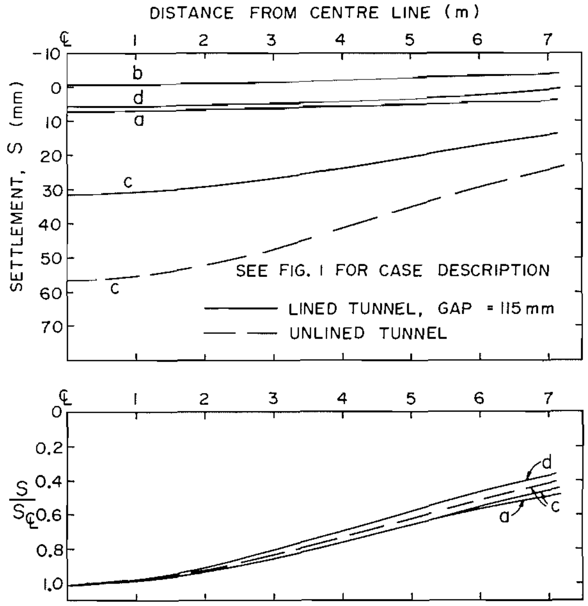

FIG. 2. Comparison of settlement profiles for the four variations in elastic modulus with depth shown in Fig. 1*a–d* (isotropic, elastic, $\nu = 0.25$).

The basic case (modulus *d* in Fig. 1) gave a larger settlement than profiles with higher or lower modulus values and the least settlement was observed for profile 1*e* which had the lowest modulus. This situation arises from the fact that low modulus values give large elastic strains which dominate over the plastic strains. Since Poisson's ratio is less than one half, the stress relief due to tunnel excavation gives rise to a volume increase (i.e., elastic expansion) in critical regions above the tunnel thereby decreasing the surface movement required to ensure compatible deformations. The stiffer elastic profile significantly reduces movements within the elastic regions; however, because the elastic strains are also small within the plastic regions the plastic strain

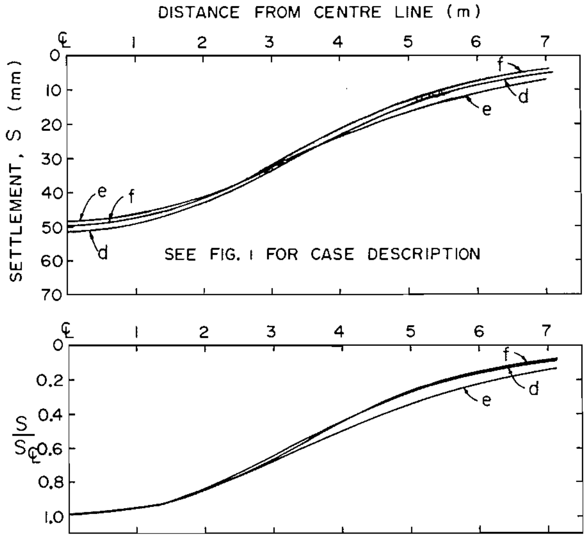

FIG. 3. Effect of modulus variations for profiles shown in Fig. 1*d*–*f*. Plasticity parameters as for case 1 in Table 2.

dominates. Since the flow rule adopted implies plastic deformation at constant volume, the dominance of plastic strains gives rise to higher surface settlement and a narrower settlement trough than would be obtained elastically.

The combination of reasonable and increasing modulus values below the tunnel with the fact that compressive stresses develop between the lining and the soil once the gap has closed reduces the calculated heave at the invert of the tunnel to a relatively small value, typically between 1 and 2 mm. This heave is not inconsistent with the observed deformations and is in marked contrast to the large heave predicted from an elastic analysis.

The foregoing results involved variations in modulus within both the silty sand and the silty clay (see Table 2). Analyses were also performed for different modulus profiles in the sand but the same modulus profile in the silty clay. Again the case corresponding to the intermediate modulus value with a linear variation between the peat and clay layers gives rise to the largest settlement. The assumption of a homogeneous modulus within the sand (be it high or low) decreased the settlement and gave a wider settlement trough than the linear variation in modulus.

It is of considerable interest to note that for this lined tunnel with a relatively small gap there is in fact only a small variation in settlement profile for a wide range of elastic modulus profiles. The elastic profile that gives the largest settlements will depend upon the size of the gap. Increasing the gap eases the restrictions upon possible compatible deformations, thereby increasing the effect of different modulus variations.

Effect of anisotropic elastic parameters

It is generally recognized (e.g., Oda 1972; Lo *et al.* 1977; Ladd *et al.* 1977) that most natural soils exhibit some degree of intrinsic anisotropy. The level of anisotropy will depend upon the geological history of the soil and will be influenced by the depositional history, erosion, weathering, and tectonic stresses. A study of available experimental data relating to the anisotropic elastic properties of soft soils suggests typical values of ν_h, ν_{hv}, and ν_{vh} ranging between 0 and 0.25, and E_h/E_v ranging between 0.5 and 1. A number of combinations of these parameters are shown in Table 1.

Elastic and elasto-plastic analyses were performed for the range of anisotropic parameters indicated in Table 1 and the results are shown in Figs. 4 and 5. These analyses showed that decreasing Poisson's ratio led to a slight increase in calculated settlement and marginally improved the shape of the settlement trough. The effect of the horizontal modulus upon the settlement trough is somewhat more complex. The value of the horizontal modulus affects both elastic strains and the stress distributions. In elastic analyses, reducing E_h/E_v marginally narrowed the calculated settlement trough. Since the stress distribution influences the extent of plastic failure, decreasing the ratio of horizontal to vertical modulus does not have the same effect upon the shape of the settlement trough calculated from an elasto-plastic analysis as it does with an elastic analysis. The narrowest settlement trough for elasto-plastic analyses (with plastic parameters for case 1 in Table 2) was obtained for $E_h/E_v = 1$. However, it should be noted that the effects of modulus ratio, modulus profile, angle of friction ϕ, and K_0 are strongly interrelated and the effect of

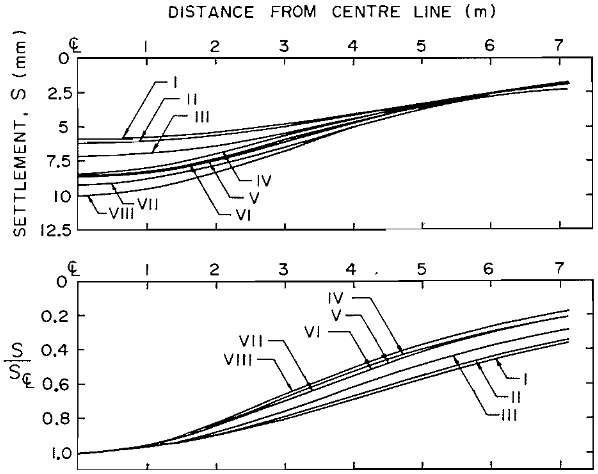

FIG. 4. Effect of elastic anisotropy for an elastic analysis. Soil profile as shown in Fig. 1*d*. Anisotropic parameters as in Table 1.

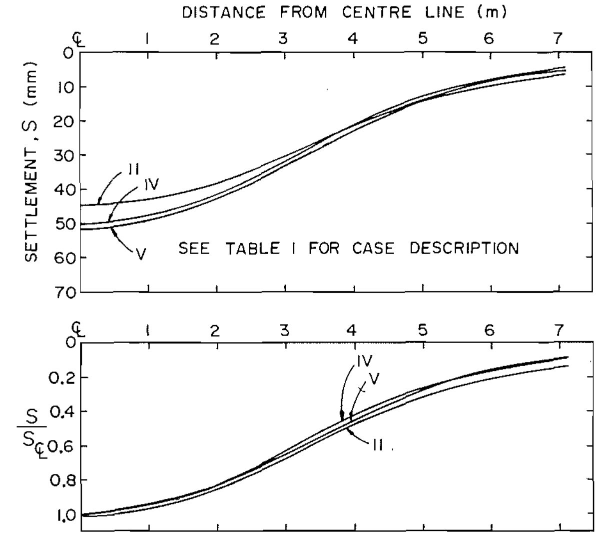

FIG. 5. Effect of anisotropic elastic parameters for an elasto-plastic analysis. Anisotropic parameters as for cases II, IV, V in Table 1. Soil profile as shown in Fig. 1*d* with plasticity parameters as for case 1 in Table 2.

changing two or more parameters will not necessarily be the sum of the individual effects.

Relatively little is known about the ratio of the independent shear modulus, G_{vh}, to the vertical modulus, E_v, for soft soils. Experimental data relating to the independent shear modulus of soft soils are scarce. Furthermore, what little data there are have been largely deduced from values of "Young's modulus" determined for samples cut at different orientations to the vertical. Since these samples were isotropically consolidated prior to testing, the relevance of the deduced shear modulus for soils with K_0 not equal to unity must be questioned.

The ratio of independent shear modulus to vertical modulus (G_{vh}/E_v) is the most uncertain and potentially the most significant anisotropic elastic parameter. Decreasing the independent shear modulus (in the range $0.25 \leq G_{vh}/E_v \leq 0.4$) increased the maximum calculated settlement and narrowed the shape of the settlement trough. The normalized settlement trough from an elasto-plastic analysis was slightly narrower than that obtained from an elastic analysis.

Effect of K_0 in the clay layer

The initial horizontal stress (and hence K_0) in the soil layers above the tunnel is of some significance insofar as it influences the extent of plastic failure within these layers. However, the critical horizontal stress is that in the clay layer surrounding the tunnel since this appreciably influences the stress changes in the soil during excavation. To illustrate the effect of K_0 in the clay layer, analyses were performed for three cases (see cases 1, 2, 3 in Table 2) with the same plasticity parameters but with $K_0 = 0.6$, 0.5, and 0.7 in the soft clay layer. The results from these analyses are shown in Fig. 6. Decreasing the value of K_0 gives an increased displacement with a 12% variation in maximum settlement for K_0 varying between 0.5 and 0.7. For this lined tunnel, the variation in K_0 between 0.5 and 0.7 had no significant effect upon the shape of the settlement trough. The effect of K_0 upon the shape of the settlement trough increases with increasing gap.

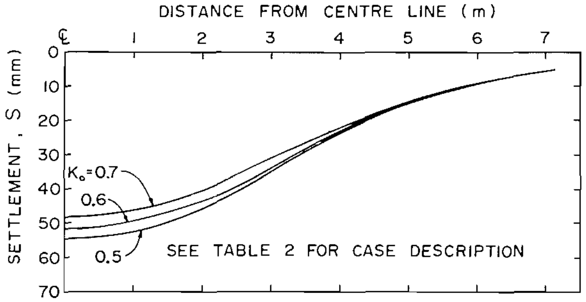

FIG. 6. Effect of K_0 in clay layer profile as shown in Fig. 1*d* for cases 1, 2, 3 in Table 2.

Effect of plasticity parameters

The critical plasticity parameters for an elasto-plastic analysis of the lined tunnels are those for the soft soil adjacent to and above the tunnel. The parameters in soil well below the invert are not critical since no failure occurs in this region.

The effect of varying the plasticity parameters c', ϕ', and K_0 (it is assumed here that K_0 is a function of ϕ) in the clay layer (see Table 2 for definition of assumed soil profile) is indicated by the settlement troughs shown in Fig. 7. Increasing the drained cohesion of the soft clay near the tunnel decreases the plastic failure (for constant ϕ) and correspondingly decreases settlement, as may be appreciated by comparing the results shown for case 1 ($c' = 0$, $\phi' = 22°$) and case 6 ($c' = 15$ kPa, $\phi' = 22°$).

Of greater interest is the influence of the angle of friction ϕ. The relationship between ϕ and K_0 complicates the effect of changing ϕ since this usually implies a different K_0. Increasing the assumed ϕ of the clay between 18° and 26° (K_0 between 0.7 and 0.55) increases the centreline settlement and gives a narrower settlement trough. The general effect upon centreline settlement might have been anticipated from the results given in Fig. 6; however, very little variation in the shape of the settlement trough was observed for variations in K_0 at constant ϕ.

Increasing the angle of friction decreases the lateral extent of plastic failure within the clay. This is further enhanced by a decrease in K_0 which may be consistent with an increase in ϕ. Since the lateral extent of soil movement into the gap around the lining is restricted,

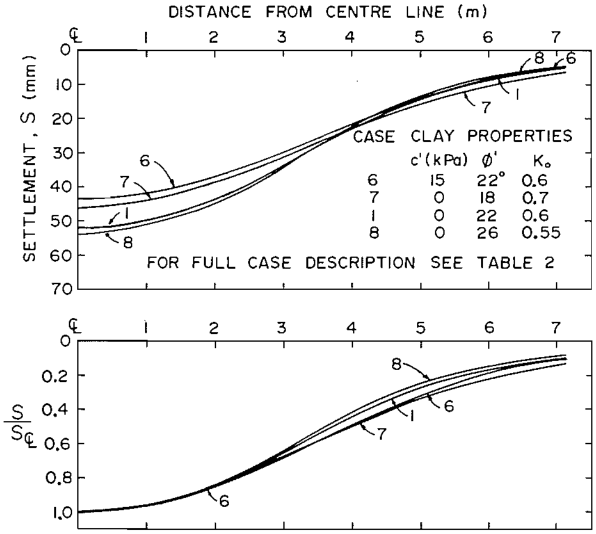

FIG. 7. Effect of c', ϕ', and K_0 in the clay layer for cases 1, 6, 7, 8 in Table 2. Profile as shown in Fig. 1*d*.

the flow rule requires that there be a higher vertical deformation within this limited plastic zone. This then gives rise to higher settlement near the centreline and lower settlement away from the centreline. This is best illustrated by the fact that the settlement near the centreline for $\phi = 18°$, $K_0 = 0.7$ (case 7 in Fig. 7) is less than that for $\phi = 22°$, $K_0 = 0.7$ (case 3 in Fig. 6) although the settlements at more than 3 m from the centreline are greater.

It is noted that increasing the elastic modulus decreases the lateral elastic strains around the tunnel and thereby further enhances the effect of increasing angle of friction ϕ.

The plasticity properties of the silty sand, although important, are less significant than those of the clay. The settlement troughs obtained for $\phi = 27°$, 30°, and 33° ($K_0 = 0.55$, 0.5, and 0.45) in the silty sand (see cases 1, 4, and 5 in Table 2) were very similar in terms of both the magnitude of the settlement and the shape of the settlement trough. The lower value of ϕ ($\phi = 27°$) gave a slightly smaller settlement near the tunnel than that obtained for $\phi = 30°$ or 33°.

Effect of gap

The gap parameter is the most critical and difficult parameter to determine. The choice of the gap to be used in an analysis will depend upon the tunnelling machine and lining diameters, soil type, and quality of workmanship. If a full face tunnelling machine is used in conjunction with good construction procedure then the loss of ground will be approximately two-dimensional and, as a lower limit, may be estimated as the difference between the maximum external diameter of the tunnelling machine and the outside diameter of the lining. Lining flexibility may permit additional movements. This can be incorporated directly in a flexible lining analysis or may be approximately included in a rigid analysis by adding the estimated crown deflection of the lining (under overburden loading conditions) to the gap. In practice, the gap will also depend on other less tangible factors such as the experience and skill of the machine operator. Depending on the design and operation of the tunnelling machine, there will be additional lost ground due to three-dimensional movements into the face, overcutting caused by pitching and yawing of the tunnelling machine, and consolidation of any disturbed or remoulded region around the tunnel caused by construction difficulties.

In design, the actual gap that will apply to a proposed tunnel will never be precisely known. However, on the basis of experience with different equipment and soil conditions, some indication of the likely range of the gap parameter can be obtained provided that there is not an excessive loss of ground due to poor workmanship or ground conditions (e.g., running ground). (It follows that the gap may be more readily bracketed when the tunnel is located in clay than when it is in loose silts or sands.)

Once the range of the gap parameter has been determined, the analysis provides a useful tool for estimating the likely magnitude and distribution of the resultant surface settlement. Even in situations where it may be very difficult to estimate the gap parameters, the analysis may still be useful for assessing the effects of variation in gap and for showing how critical this factor may be.

To give some indication of the effect of the gap, Fig. 8

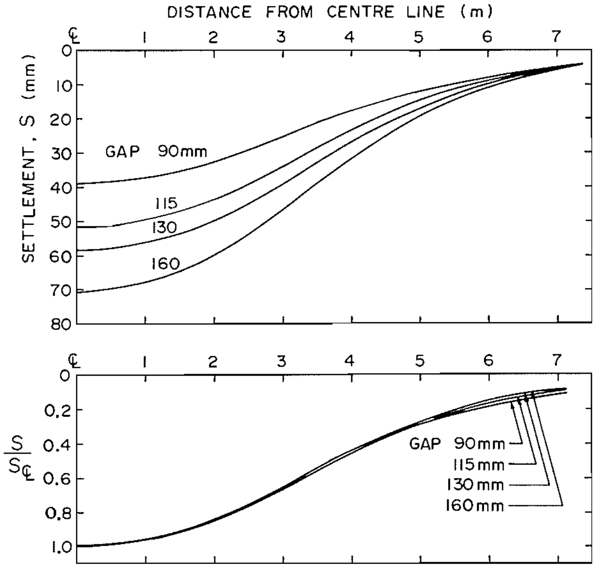

FIG. 8. Effect of gap (no grout injection). Soil profile as shown in Fig. 1*d*. Plasticity parameters as for case 1 in Table 2.

shows the settlement troughs for assumed gaps of 90, 115, 130, and 160 mm. The gap of 90 mm corresponds to the difference between the assumed external diameter of the tunnelling machine and the lining. Increasing the gap (i.e., the loss of ground) appreciably increases the volume of the settlement trough. Increasing the gap also magnifies the effect of other parameters such as the modulus profile, the plasticity parameters, and K_0.

Effect of grout pressure

The injection of clay grout into the tailpiece void after erection of the lining may be expected to decrease settlements. However, much of the soil movement will have occurred prior to the grout injection. As a first approximation, it is assumed here that the initial soil deformations due to the excavation and installation of the lined tunnel occur prior to clay grout injection.

The effect of clay grout injection at a pressure of 170 kPa is illustrated by the settlement trough shown in Fig. 9 for gaps of 115, 130, and 160 mm. Comparing the results in Figs. 8 and 9, it is seen that the grout decreases the surface settlement and leads to a slightly narrower settlement trough.

Grout injection may be an important factor to be considered if the construction procedures are conducive to effective grouting. However, even in these cases this study confirms the observed fact that injection of grout into the tailpiece void may reduce but will not prevent surface settlement. Furthermore, in many practical situations, problems will arise that prevent effective grouting, and the influence of the grout may be neglected. (The effects of compressed air and grouting have been discussed in greater detail by Rowe and Kack (1983).)

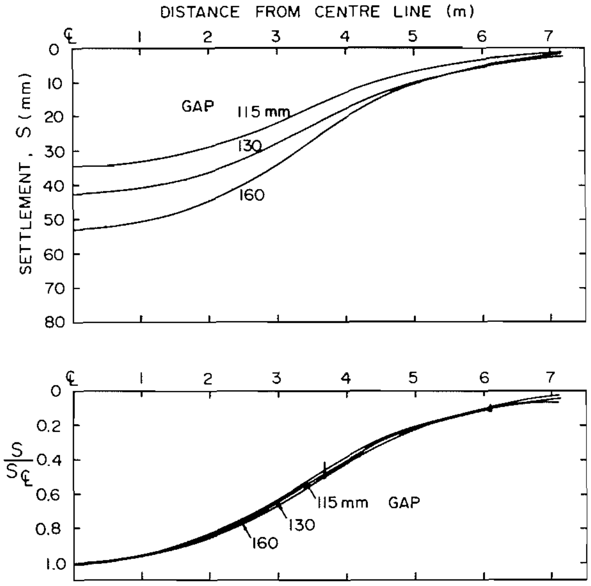

FIG. 9. Effect of gap (grout pressure = 170 kPa). Soil profile as shown in Fig. 1*d*. Plasticity parameters as for case 1 in Table 2.

Effect of soil unit weight

In the preceding analyses, it has been assumed that the soil has a unit weight (γ = 14.7 kN/m^3). The unit weight is an important parameter in two respects. Firstly, the amount of stress redistribution within the soil due to the tunnel excavation will increase with increasing unit weight of the soil above the tunnel invert and this will tend to increase settlement. Secondly, increasing unit weight influences the mean stress in the soil and may increase the shear strength of the soil, thereby tending to reduce settlement. To illustrate these effects, analyses were performed for γ = 14.7 kN/m^3 (the basic case) and γ = 19 kN/m^3; the resulting settlement profiles are shown in Figs. 10 and 11. For the case with γ = 19 kN/m^3, a moderate gap (115 mm), and no grout injection (Fig. 10), the stress redistribution due to excavation does not counteract the higher shear strength (compared to that for γ = 14.7 kN/m^3) and results in less plasticity, less settlement, and higher lining pressures than those calculated for the basic case. However, for the same reasons, grout injection does not have as much effect for γ = 19 kN/m^3 as it does when γ = 14.7 kN/m^3 and hence the calculated settlement with grout injection is greatest for γ = 19 kN/m^3. This demonstrates the symbiotic relationship between strength parameters, initial stresses, and the gap parameter and is further illustrated by reference to Fig. 11 which gives the corresponding settlement profiles calculated

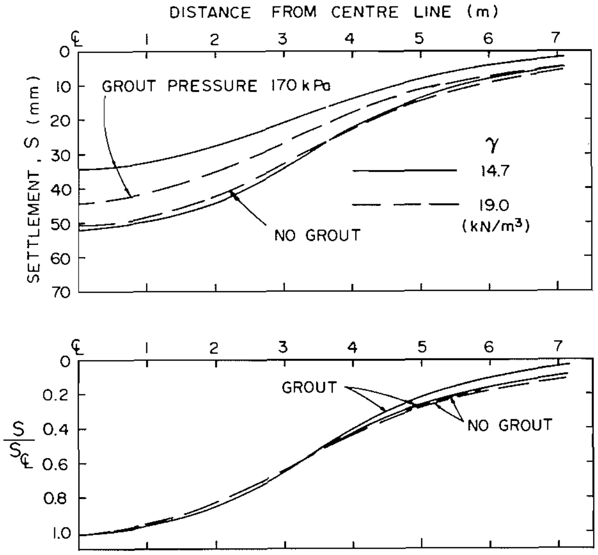

FIG. 10. Effect of unit weight for a gap of 115 mm. Soil profile as shown in Fig. 1*d*. Plasticity parameters as for case 1 in Table 2.

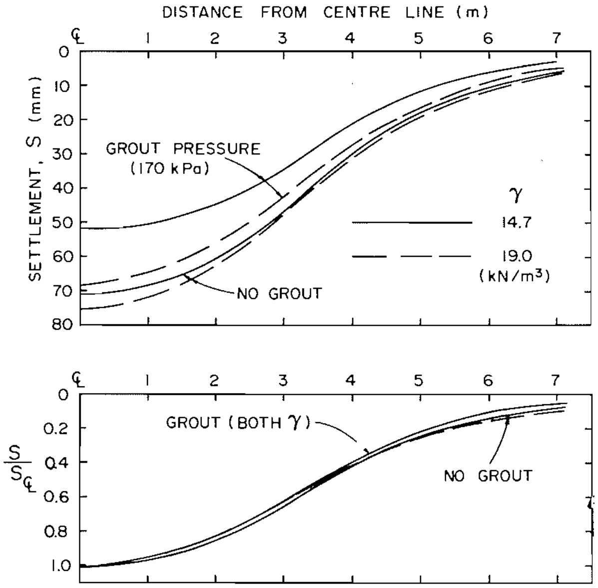

FIG. 11. Effect of unit weight for a gap of 160 mm. Soil profile as shown in Fig. 1*d*. Plasticity parameters as for case 1 in Table 2.

for a gap of 160 mm. In this case the stress redistribution allowed by the larger gap does counteract the higher initial shear strength of the soil for $\gamma = 19\,\mathrm{kN/m^3}$ and gives rise to larger settlements than calculated for $\gamma = 14.7\,\mathrm{kN/m^3}$ (no grout). As previously, grout injection also has less effect for the case with $\gamma = 19\,\mathrm{kN/m^3}$ and hence the settlement is again greater than that for $\gamma = 14.7\,\mathrm{kN/m^3}$.

Conclusions

A technique suitable for estimating the deformations caused by the installation of a lined tunnel in soft ground has been described. This technique is considered most appropriate for tunnels constructed using a tunnelling machine.

The method of analysis was used in a parametric study to identify potentially significant factors affecting the prediction of settlement due to tunnelling in soft soil. A number of conclusions may be drawn from this investigation.

1. The gap parameter used in the plane strain analysis is the most critical and difficult parameter to determine. The choice of gap will depend on tunnelling machine and lining characteristics, soil type, and the experience and skill of the tunnelling machine operator.

Thus it is impossible to precisely determine the gap parameter prior to construction. What can be assessed, however, is the effect of a reasonable variation in the gap upon the surface settlement. Thus, limited parametric studies conducted at the design stage can show how critical the gap really is for a particular design and hence may be very useful in deciding the technical factors to be incorporated into the specifications. It should be recognized that the gap may be more readily estimated for tunnels passing through clays than for tunnels constructed in loose silts and sands where major ground loss due to running ground may occur.

2. In the analysis, stress relief due to tunnelling will give rise to a predicted heave of the underlying soil.

If the correct soil parameters are adopted, this predicted heave will be small (as it usually is in practice). However, careful attention must be given to selection of an appropriate modulus for the soil below the tunnel. The adoption of a modulus that is too low will result in a large heave and an underestimation of the surface settlement.

3. Elastic analyses will not provide a good estimate of surface settlements due to construction of lined tunnels in soft soils if, as is usually the case, there is significant plastic failure within the soil. In these cases the prediction of surface settlement requires consideration of the plastic flow characteristics of the material and appropriate modelling of the principal stress rotations that occur above the tunnel.

4. It is important to recognize that the stress path experienced by the soil during tunnel excavation is quite different from that experienced by soil beneath a foundation. Elastic "moduli" determined from either drained or undrained compression tests may not be appropriate. Rather, the modulus of the soil is closer to the extension modulus which may be considerably greater than the compression modulus. (Experimental justification for this with regard to the Thunder Bay tunnel has been given by Lo and Rowe (1982).) In the analysis, it is not necessarily conservative to adopt the compression modulus. For soft soils and a small to moderate gap, the predicted settlement may be greater and the settlement trough narrower if the extension modulus is adopted because this permits more plastic strain to occur.

5. For cases in which there is only a moderate loss of ground, there may be only a small variation in predicted settlements for a wide range of assumed elastic modulus profiles. In the examples in this paper the lining and plasticity govern the soil response and under these circumstances the detailed determination of the elastic modulus profile may not be necessary.

6. Decreasing the coefficient of lateral earth pressure at rest (K_0) in the soil surrounding the tunnel tends to increase the magnitude of surface settlement but does not greatly affect the shape of the settlement trough when there is only moderate loss of ground.

7. In predicting the maximum settlement and distortion due to the installation of a lined tunnel in soft soil, it is not necessarily conservative to adopt a low angle of friction. Higher angles of friction for clay tend to give

higher maximum settlements and a narrower settlement trough if it is assumed that K_0 is related to the angle of friction by typical correlations.

8. Injection of clay grout into the tailpiece void may reduce but generally will not prevent surface settlement.

9. The unit weight of the soil above the tunnel invert may significantly influence the surface settlement. However, the effect of unit weight is not immediately obvious. Depending on the strength and gap parameters, increasing unit weight may either decrease or increase the estimated surface settlement.

It is not suggested that this study has been exhaustive; however, it is considered that the results presented in this paper do provide insight into the possible effects of different parameters upon predicted subsidence. This insight is important in the determination of relevant soil parameters for use in analysis and in selecting the range of cases to be examined in design situations. The application of this theory and conclusions to the analysis of tunnel behaviour for six case histories has been described by Rowe and Kack (1983) and Lo and Rowe (1982).

Acknowledgements

The work described in this paper was primarily supported by grant No. A1007 from the Natural Sciences and Engineering Research Council of Canada. Additional funds, in the form of an award to Mr. G. J. Kack, were supplied by the Ontario Concrete Pipe Association. The associated field investigation is being supported by grant No. 31155 from the Department of Supply and Services Canada and the National Research Council of Canada. The authors are also grateful to Mr. K. M. Lee for his assistance in the preparation of some of the figures.

ATTEWELL, P. B., and FARMER, I. W. 1974. Ground deformations resulting from shield tunnelling in London Clay. Canadian Geotechnical Journal, **11**, pp. 380–395.

DAVIS, E. H. 1968. Theories of plasticity and the failure of soil masses. *In* Soil mechanics selected topics. *Edited by* I. K. Lee. Butterworths, Sydney, Australia.

GHABOUSSI, J., RANKEN, R. E., and Karshenas, M. 1978. Analysis of subsidence over soft-ground tunnels. ASCE International Conference on Evaluation and Prediction of Subsidence, Pensacola Beach, pp. 182–196.

KACK, G. J. 1981. Prediction of subsidence above shallow tunnels in soft soil. M.E.Sc. thesis, University of Western Ontario, London, Ont., 380 p.

KULHAWY, F. H. 1974. Finite element modelling criteria for underground openings in rock. International Journal of Rock Mechanics and Mineral Sciences, **11**, pp. 465–472.

LADD, C. C., FOOTT, R., ISHIHARA, K., SCHLOSSER, F., and POULOS, H. G. 1977. Stress-deformation and strength characteristics. Proceedings, 9th International Conference on Soil Mechanics and Foundation Engineering, Tokyo, pp. 421–494.

LO, K. Y., and ROWE, R. K. 1982. Prediction of ground subsidence due to tunnelling in clays. Faculty of Engineering Science, Research Report GEOT-10-82, University of Western Ontario, London, Ont., 353 p.

LO, K. Y., LEONARDS, G. A., and YUEN, C. 1977. Interpretation and significance of anisotropic deformation behaviour of soft clays. Norwegian Geotechnical Institute, Publ. No. 117, Oslo, Norway.

ODA, M. 1972. Initial fabrics and their relations to mechanical properties of granular material. Soils and Foundations, **12**, pp. 1–18.

PALMER, J. H. L., and BELSHAW, D. J. 1980. Deformations and pore pressures in the vicinity of a precast, segmented, concrete-lined tunnel in clay. Canadian Geotechnical Journal, **17**, pp. 174–184.

PECK, R. B. 1969. Deep excavations and tunnelling in soft ground. Proceedings, 7th International Conference on Soil Mechanics and Foundation Engineering, Mexico City, pp. 225–290.

ROWE, R. K., and KACK, G. J. 1983. Some applications for analysis in the estimation of surface settlement caused by tunnelling in soft ground. Canadian Geotechnical Journal, **20** (in press).

ROWE, R. K., BOOKER, J. R., and BALAAM, N. P. 1978. Application of the initial stress method to soil structure interaction. International Journal of Numerical Methods in Engineering, **12**, pp. 873–880.

SCHMIDT, B. 1969. Settlements and ground movements associated with tunnelling in soil. Ph.D. thesis, University of Illinois, Urbana, IL.

TAN, T. Y., and CLOUGH, G. W. 1980. Ground control for shallow tunnels by soil grouting. Journal of the Geotechnical Engineering Division, ASCE, **106** (GT9), pp. 1037–1057.

ZIENKIEWICZ, O. C., VALIAPPAN, S., and KING, I. P. 1969. Elasto-plastic solutions of engineering problems—initial stress finite element approach. International Journal of Numerical Methods in Engineering, **1**, pp. 75–100.