Theoretical solutions for calculating leakage through composite liner systems

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ABSTRACT: A new semi-analytic solution for the leakage of fluid through a circular hole in an otherwise essentially impermeable geomembrane underlain by a clay liner is presented. This solution covers the full range of layer thickness between very thin (one-dimensional flow) and infinitely thick (Forchheimer's solution). The solution can be used for a wide range of practical problems where the radius of the hole may range from a pinhole to a large quasi-circular wrinkle in a perforated geomembrane (i.e. where the "hole" is considered to be the area where there is no contact between the geomembrane and clay). The solution assumes perfect contact between the geomembrane away from the hole but does allow consideration of hydraulic anisotropy of the clay layer. Using the proposed theory, a very simple, analytic, approximate expression is obtained. This solution can be used in hand calculations to establish the leakage rate in many practical design situations.

1 INTRODUCTION

Composite liner systems involving a geomembrane (e.g. high density polyethylene) over a clay liner (e.g. compacted clay, CCL, or a geosynthetic clay liner, GCL) play an important role in the design of modern waste and other fluid impoundment facilities. The intact geomembrane provides an essentially impermeable barrier to water. Thus, leakage of water through the composite liner will be controlled by the number and size of holes in the geomembrane. These holes can often be approximated as circular and a number of limiting solutions have been obtained for a circular hole overlying a clay layer. For example, if one assumes that there is perfect contact between the geomembrane outside the hole then the flow Q through a hole on a semi-infinite isotropic soil deposit is given by Forchheimer's (1930) Equation

$$Q = 4 r_o h_d k \tag{1}$$

where r_o is the radius of the hole, k is the hydraulic conductivity of the clay liner, and h_d is the head loss and is generally taken to be the height of fluid above the hole.

An alternative approach, adopted by Jayawickrama et al. (1988), was to consider a transmissive zone between the geomembrane and the clay. They developed a solution that considered axi-symmetric horizontal radial flow at the interface and 1-D vertical flow through the clay between the transmissive layer below the geomembrane and a transmissive layer below the clay layer. They assumed that the head in the underlying transmissive layer was at the level of the geomembrane (i.e. h_a = D in Figure 1). This solution has been widely used (e.g. Giroud & Bonaparte 1989; Giroud et al. 1994, 1998) in practice. However, as useful as they are, both solutions have limitations. Equation 1 is limited to very deep layers; a situation generally not encountered. The solution developed by Jayawickrama et al. (1988) considers the case of a layer of finite depth but breaks down where the transmissivity of the interface between the geomembrane and liner is very low (e.g. when there is good contact as considered numerically by Walton and Sager (1990) and Walton et al. (1997)).

The objective of this paper is to provide a relatively simple solution methodology for a number of additional cases that may be of practical significance. In particular, attention is focussed on the case of a geomembrane in good contact with the soil for $r > r_o$ (i.e. no transmissive layer between the geomembrane and the soil) and a layer of finite depth. No restriction is placed on the size of the hole that may be either small or large relative to the thickness of the clay layer. This has important potential applications for holes in a geomembrane over a GCL liner (that may be only 5-10 mm thick) as shown schematically in Figure 1a or when simulating an approximately circular perforated wrinkle above a compacted clay liner as shown schematically in Figure 1b.

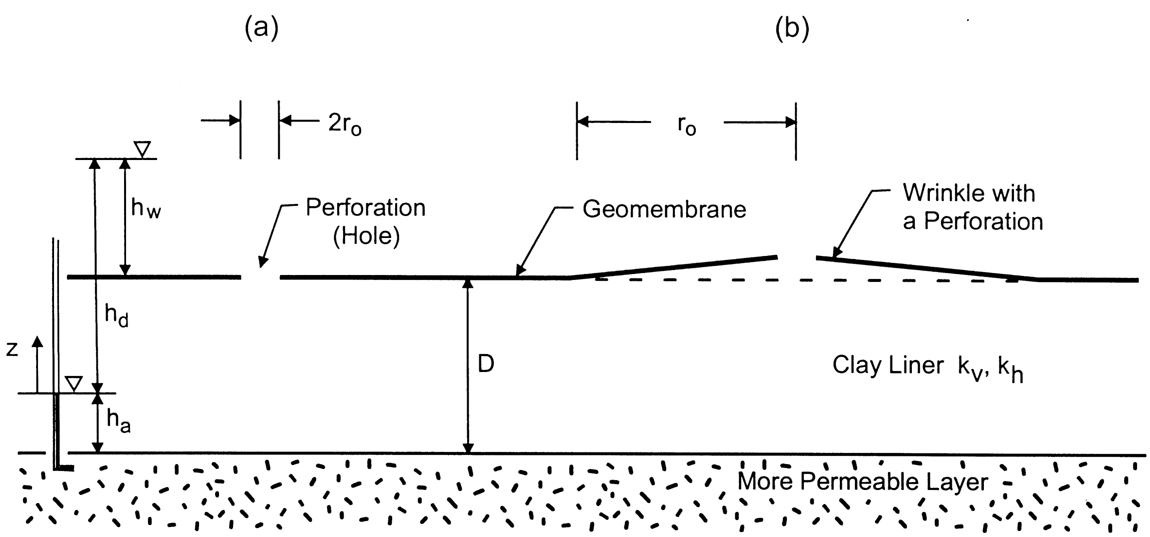

Figure 1. Problem definition showing (a) hole of radius r_o in geomembrane in intimate contact with clay liner and (b) wrinkle with perforation - effective "hole" radius r_o is for zone where geomembrane is not in intimate contact with the clay liner (GCL or CCL). Note: h_w = fluid (e.g. leachate) head on geomembrane; D = thickness of clay liner; k_v = vertical hydraulic conductivity of clay liner; k_h = horizontal hydraulic conductivity of clay liner; h_a = potentiometric level in permeable layer relative to bottom of clay liner; $h_d = D - h_a + h_w$ = head drop across composite liner.

2 THEORY - GENERAL CASE

Considering the general case shown in Figure 1, we seek a solution to the mixed boundary value problem associated with the specified head at the hole

$$h = h_d = (D - h_a) + h_w \quad 0 \le r \le r_o \qquad \text{at the surface } (z=D-h_a) \tag{2}$$

and specified zero vertical flux where there is intact geomembrane

$$v_z = k_v \frac{dh}{dz} = 0 \quad r > r_o \qquad \text{at the surface } (z=D-h_a) \tag{3}$$

and k_v is the vertical hydraulic conductivity. Suppose that the velocity is not known but that the total head, $h_o(x,y)$ is known in some region R, and assume that the velocity $v_o(x,y)$ (at $z=D-h_a$) on R can be approximated by an expression of the form

$$v_o(x,y) = c_o\phi_o(x,y) + \ldots c_n\phi_n(x,y) \tag{4}$$

where we will seek basis functions ϕ_j and coefficients c_j that will provide the best approximation to the actual velocity distribution.

A solution can be achieved in transform space and so introducing the Fourier transforms gives

$$h(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(\alpha x+\beta y)} H(\alpha,\beta,z)\,d\alpha\,d\beta \tag{5a}$$

$$H(\alpha,\beta,z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(\alpha x+\beta y)} h(x,y,z)\,dx\,dy \tag{5b}$$

$$v(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(\alpha x+\beta y)} V(\alpha,\beta,z)\,d\alpha\,d\beta \tag{6a}$$

$$V(\alpha,\beta,z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(\alpha x+\beta y)} v(x,y,z)\,dx\,dy \tag{6b}$$

where

$$V(\alpha,\beta,z) = -k_v \frac{\partial H(\alpha,\beta,z)}{\partial z}$$

For a horizontally layered deposit it is easy to establish (see Appendix A) that

$$H(\alpha,\beta,z) = L(\alpha,\beta,z)V_o(\alpha,\beta) \tag{7}$$

where

$$V_o = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(\alpha x+\beta y)} v_o(x,y)\,dxdy$$
$$= \frac{1}{4\pi^2}\iint_R e^{-i(\alpha x+\beta y)} v_o(x,y)\,dxdy \qquad (8)$$

(where v_o=0 outside R). For steady state conditions and a layer of thickness D underlain by a permeable base it can be shown (Appendix A) that:

$$L(\alpha,\beta,z) = \frac{\sinh\left[\rho(h_a+z)\sqrt{\dfrac{k_h}{k_v}}\right]}{\rho\sqrt{k_h k_v}\cosh\left[\rho D\sqrt{\dfrac{k_h}{k_v}}\right]} \qquad (9)$$

and for z = D-h_a (i.e. at the top of the layer)

$$L_o(\alpha,\beta) = \frac{\tanh\left[\rho D\sqrt{\dfrac{k_h}{k_v}}\right]}{\rho\sqrt{k_h k_v}} \qquad (10a)$$

and for $D \to \infty$,

$$L_o(\alpha,\beta) = \frac{1}{\rho\sqrt{k_h k_v}} \qquad (10b)$$

where $\rho = \sqrt{\alpha^2+\beta^2}$.

It follows from Eqs. 4 and 8 that

$$V_o(\alpha,\beta) = \sum_{j=0}^{n} c_j \Phi_j(\alpha,\beta) \qquad (11)$$

where

$$\Phi_j = \frac{1}{4\pi^2}\iint_R e^{-i(\alpha x+\beta y)}\phi_j\,dxdy \qquad (12)$$

Now from Eqs. 5a, 7 and 11,

$$h(x,y,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}L(\alpha,\beta,z)V_o(\alpha,\beta)e^{i(\alpha x+\beta y)}\,d\alpha d\beta$$

$$=\sum_{j=0}^{n}c_j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}L(\alpha,\beta,z)\Phi_j(\alpha,\beta)e^{i(\alpha x+\beta y)}d\alpha d\beta \tag{13}$$

and at z=D-h_a (top of the layer)

$$h_o(x,y)=\sum_{j=0}^{n}c_j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}L_o(\alpha,\beta)\Phi_j(\alpha,\beta)e^{i(\alpha x+\beta y)}d\alpha d\beta \tag{14}$$

and hence

$$\int\int_R h_o\phi_\ell(x,y)dxdy=\sum_{j=o}^{n}c_j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[L_o(\alpha,\beta)\Phi_j(\alpha,\beta)\int\int_R e^{i(\alpha x+\beta y)}\phi_\ell(x,y)dxdy]d\alpha d\beta$$

$$=4\pi^2\sum_{j=0}^{n}c_j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi_\ell^*L_o(\alpha,\beta)\Phi_j d\alpha d\beta \tag{15}$$

where Φ_ℓ^* is the complex conjugate of Φ_ℓ.

Hence,

$$\sum A_{\ell j}c_j=b_\ell \tag{16}$$

for a given set of basis functions ϕ_j; this can be solved for the coefficients c_j that provide the best approximation to the velocity distribution where

$$A_{\ell j}=4\pi^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi_\ell^*L_o(\alpha,\beta)\Phi_j d\alpha d\beta \tag{17}$$

$$b_\ell=\int\int_R h_o(x,y)\phi_\ell(x,y)dxdy \tag{18}$$

3 SOLUTION FOR A CIRCULAR HOLE ON A HALF-SPACE

To illustrate the approach for a problem with a known solution, consider the special case of a circular hole on a half-space and suppose

$$\phi_o = \left(1 - \frac{r^2}{r_o^2}\right)^{-0.5} \qquad (19)$$

and hence from Eq. 4

$$v_o = c_o[1 - \frac{r^2}{r_o^2}]^{-1/2} \qquad 0<r<r_o \qquad (20a)$$

$$= 0 \qquad r \geq r_o \qquad (20b)$$

then (after integration) it follows from Eq. 12

$$\Phi_o = \frac{1}{2\pi}\frac{r_o}{\rho}\sin\rho r_o \qquad (21)$$

and hence from Eqs. 10b and 14,

$$h_o(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_o \frac{e^{+i(\alpha x+\beta y)}}{\rho\sqrt{k_h k_v}} \frac{r_o \sin\rho r_o}{2\pi\rho} d\alpha d\beta$$

$$= \frac{c_o r_o}{\sqrt{k_h k_v}} \int_o^{\infty} J_o(\rho r) \frac{\sin\rho r_o}{\rho} d\rho$$

$$= \frac{c_o r_o}{\sqrt{k_h k_v}} \frac{\pi}{2} \qquad 0 < r < r_o \qquad (22a)$$

$$= \frac{c_o r_o}{\sqrt{k_h k_v}} \sin^{-1}\left(\frac{r_o}{r}\right) \qquad r_o < r < \infty \qquad (22b)$$

Hence, since $h_o(x,y) = h_d$ is a constant for $0<r<r_o$, from Eq. 22a

$$c_o = \frac{2h_d\sqrt{k_h k_v}}{\pi r_o}$$

and the flow Q is given by

$$Q = 2\pi c_o \int_o^{r_o} r(1-\frac{r^2}{r_o^2})^{-0.5} dr = 2\pi r_o^2 c_o$$

$$= 4r_o \, h_d \sqrt{k_h k_v} \tag{23}$$

that reduces to Forchheimer's (1930) solution for an isotropic soil with $k=k_h=k_v$ (see Eq. 1). Thus in this case only one term is required in Eq. 4 to obtain the exact solution.

4 SOLUTION FOR A CIRCULAR HOLE ON A LAYER OF FINITE DEPTH

The functions ϕ_0, ϕ_1,ϕ_j etc. in Eq. 4 provide the basis for mixed boundary value problems through the functions Φ_0, Φ_1,…,Φ_j given by Eq. 12. Consider basis functions of the form

$$\phi_o = (1-\frac{r^2}{r_o^2})^{-1/2} \tag{24a}$$

$$\phi_\mu = (r_o^2 - r^2)^{\mu-1} \qquad \mu = 1,2,.... \tag{24b}$$

In general, one could use as many basis functions as one wishes, and the coefficients c_j will eliminate those not needed. However, it was found that for all cases examined, three basis functions ϕ_o, ϕ_1 and ϕ_2 were sufficient.

Thus, it follows from Eq. 12 that for these axi-symmetric conditions,

$$\Phi_o = \frac{r_o}{2\pi} \frac{\sin(\rho r_o)}{\rho} \tag{25a}$$

$$\Phi_\mu = \frac{2^{\mu-1}}{2\pi} \frac{r_o^\mu \Gamma(\mu)}{\rho^\mu} J_\mu(\rho r_o) \qquad \mu = 1,2.... \tag{25b}$$

where $\Gamma(\mu) = (\mu-1)!$ and $\Gamma(1) = 1$ and hence the coefficients $A_{\ell,j}$ ($\ell = 0,1,2$; $j = 0,1,2$) can be calculated numerically from Eq. 17.

The values of b_ℓ are given by Eq. 18.

$$b_o = 2\pi r_o^2 h_d \tag{26a}$$

$$b_\mu = \frac{\pi r_o^{2\mu} h_d}{\mu} \qquad \mu = 1,2,.... \tag{26b}$$

for a constant head $h_o(x,y) = h_d = D-h_a+h_w$. Once the coefficients $A_{\ell j}$ and b_ℓ are evaluated, Eq. 16 can be solved for the constant c_j that match the head distribution at the hole and hence the velocity distribution can be deduced from Eq. 11. Finally, the flow can be evaluated.

$$Q = \int\int_R v_o(x,y)dxdy \tag{27}$$

and from Eqs. 4

$$Q = \int\int_R \sum_{j=0}^{n} c_j \phi_j \, dxdy = \sum_{j=0}^{n} c_j \int\int_R \phi_j dxdy \tag{28}$$

Hence, from Eqs. 18 and 28

$$Q = \sum_{j=0}^{n} \frac{c_j b_j}{h_d} \tag{29}$$

for constant head $h_d = h_w + D-h_a$ over R, where b_j is known from Eq. 26 and c_j are known by solving Eq. 16.

5 NUMERICAL CONSIDERATIONS

The solution presented above is essentially analytic, however, the number of basis functions required to converge to the exact solution is not known a priori and so the number of terms, n, may be varied. However, as noted earlier, it was found that in all situations examined, only three terms were required. For deep deposits ($r_o/D \to 0$), one term was sufficient.

The evaluation of the coefficients $A_{\ell j}$ in Eq. 17 involves numerical integration. In general, this is a two-dimensional integration, however, for the case of circular symmetry considered in this paper, Eq. 17 reduces to

$$A_{\ell j} = 8\pi^3 \int_o^\infty \Phi_\ell(\rho) L_o(\rho) \Phi_j(\rho) \rho d\rho \tag{30a}$$

where Φ_ℓ, Φ_j are given by Eq. 25 and $L_o(\rho)$ is given by Eq. 10. Note also that the infinitely deep case ($r_o/D \to 0$) arises naturally from Eq. 10a that reduces to Eq. 10b in a numerically stable manner. The integration required in Eq. 30 can be readily performed by Gauss Quadrature and is approximated by

$$A_{\ell j} = 8\pi^3 \int_o^W \Phi_\ell(\rho) L_o(\rho) \Phi_j(\rho) \rho d\rho \tag{30b}$$

where the width W can be increased until $A_{\ell j}$ converges to a unique value.

Due to the fact that there is no discretization error (i.e. unlike finite element or finite difference solutions), this approach readily allows considering a wide range of combinations of layer thickness D and hole size r_o.

6 LIMITING CASES

As previously noted, for the limiting case of a deep layer ($r_o/D \rightarrow 0$) is given by Eq. 23

$$Q = 4 r_o h_d \sqrt{k_h k_v} \qquad (23)$$

and for a very shallow layer ($r_o/D \rightarrow \infty$) the solution becomes one-dimensional and

$$Q = \pi k_v r_o^2 h_d / D = \pi \frac{r_o}{D} r_o h_d k_v \qquad (31)$$

Between these two limits the flow increases and it may be hypothesized that the solution for intermediate values of r_o/D may be given by

$$Q = \left(4 + \frac{F r_o}{D}\right) r_o h_d k_v \qquad (32)$$

where one seeks a function F that will provide a reasonable approximation for all values of $0 \leq r_o/D \leq \infty$.

Examination of limiting cases and Eq. 32 would suggest that flows through a "hole" could be presented in terms of a dimensionless flow M:

$$\frac{Q}{r_o h_d k_v} = M = \left(4 + \frac{F r_o}{D}\right) \qquad (33)$$

and it will be shown in the following section that to sufficient accuracy (better than 3%), F can be given by

$$F = 2.455 + 0.685 \tanh[0.6 \,\ell n\,(r_o / D)] \qquad (34)$$

and hence $F \rightarrow 1.77$ as $r_o/D \rightarrow 0$

and $F \rightarrow \pi$ as $r_o/D \rightarrow \infty$.

7 APPLICATION TO COMPOSITE LINER INVOLVING A GCL

Geosynthetic clay liners (GCLs) with powdered bentonite in the upper geotextile have been developed. Thus, provided that the geomembrane liner is in intimate contact with the GCL, there will be negligible horizontal flow at the interface between the geomembrane and GCL. Under these circumstances, the transmissivity, θ, of the interface between the geomembrane and GCL away from the hole is very low ($\theta \cong 0$) and the solution presented in the previous section may be used for two important practical cases. The first involves a "hole" in the geomembrane that is a physical perforation (e.g. a puncture) and the geomembrane is in intimate contact with the GCL at all points near the puncture (hole). The second involves a wrinkle with a perforation where the size of the "wrinkle" (i.e. zone of geomembrane that is not in intimate contact with the GCL) defines the size of the "hole". Holes and wrinkles that are approximately two-dimensional (e.g. waves) may be modelled as described by Rowe (1998). This present paper focuses on holes and wrinkles that are either circular or have dimensions such that they can be reasonably approximately as circular. In the case where there is a perforation in a wrinkle, the flow will be controlled by the minimum of either (a) the capacity for flow through the perforation (which is controlled by Bernoulli's equation; see Giroud & Bonaparte 1989a); or (b) the flow through the GCL in a zone where there is no intimate contact between geomembrane and GCL. Condition (a) can be readily checked on a hand calculator as described by Giroud and Bonaparte (1989a) or Rowe (1998) and is not considered further here. Condition (b) may be checked using the theory presented in this paper.

Analyses were initially performed for a GCL with a hydrated thickness of D=0.01 m, $k \sim 2\times10^{-10}$ m/s, h_w = 0.3 m and $h_d = D+h_w$. These represent typical conditions of a composite liner consisting of a geomembrane over a GCL over a secondary leachate collection (leak detection) system at relatively low applied vertical stress ($\cong$30 kPa) and allowing for the effect of clay-leachate interaction in assessing the k value (see Rowe 1998).

Results were obtained for a range of hole sizes, r_o, and the results were plotted in terms of the dimensionless flow, M, per hole versus r_o/D and the results are plotted in Figure 2. It can be seen that the deep solution (Forchheimer 1930) is only really valid (to better than 1%) for very small holes, $r_o/D<0.02$ or $r_o<0.0002$ m in this specific case, and that to better than about 5% accuracy is only valid for $r_o/D\leq0.1$ ($r_o\leq1$ mm). This corresponds to a "small hole" (i.e. perforation in the geomembrane). A large hole (as defined by Giroud and Bonaparte 1989a,b) corresponds to $r_o/D\cong0.56$ (r_o=5.64 mm) and in this case the flow exceeds that predicted by Forchheimer's Equation (Eq. 23) by 30%. Thus, for most practical cases involving perforations and wrinkles, Forchheimer's (1930) equation is not adequate. It is also evident that the solution only approaches the one-dimensional case (Eq. 31) when $r_o/D>20$ ($r_o>0.2$ m in this case). This would be relevant to a large wrinkle or puncture. Thus there is a need for an intermediate solution for a wide range of practical cases.

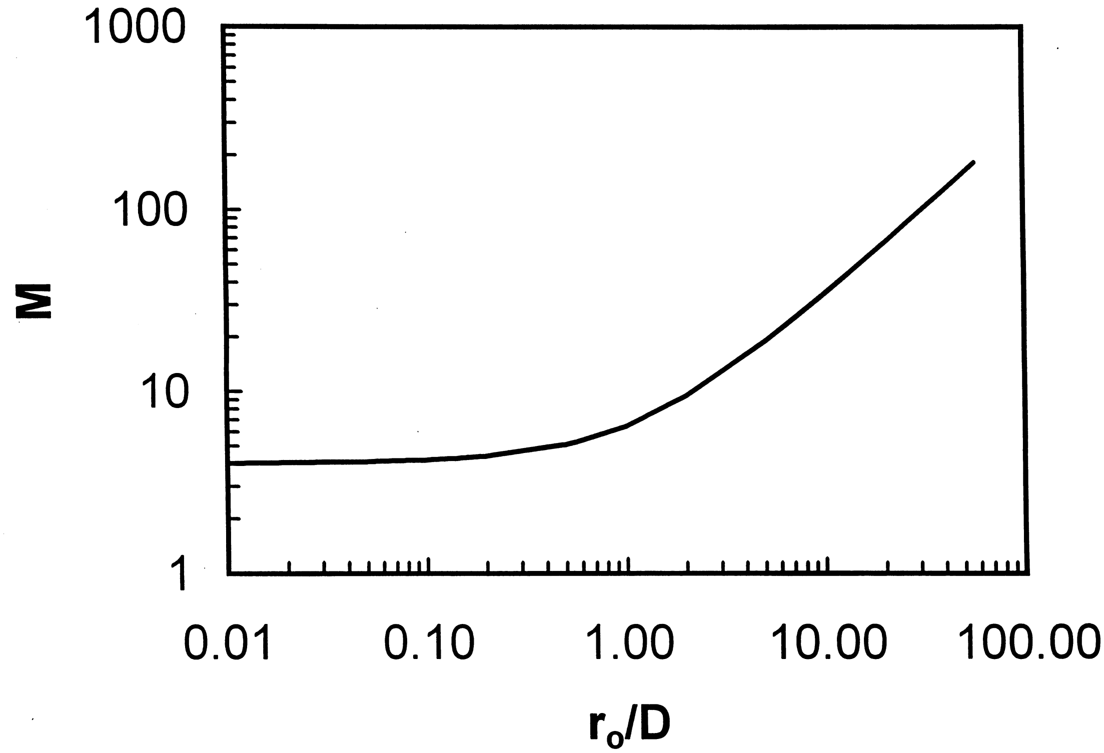

Figure 2. Variation in dimensionless flow $M=Q/(r_o h_d k_v)$ with ratio of hole size, r_o, to clay liner thickness, D.

From the results presented in Figure 2 and Eq. 33, the factor F can also be deduced and is plotted against $\ell n\ r_o/D$ as shown in Figure 3. It can be seen that there is a transition from $F \cong 1.77$ for small values of r_o/D to $F=\pi$ for large values of r_o/D. In order to find a simple approximate equation that could be used with a hand calculator, we seek a simple analytical approximation to the variation in F with r_o/D. The nature of the variation suggests a simple function of the form

$$F = 2.455 + 0.685 \tanh[w\ \ell n(r_o / D)] \qquad (35)$$

where w is a fitting parameter selected to minimize the root mean squared error of the fit to the results (shown as data points in Figure 3) obtained using the theory presented in the previous section. This was achieved for w=0.6 giving the functional form shown by the solid line in Fig. 3 and given by Eq. 34. As can be seen, this provides an excellent fit with a maximum deviation of less than 3% and typically less than 0.5%.

Equation 34 has considerable potential for use in practical engineering calculations, however, it is necessary to establish how well it works for situations other than the specific case for which it was established. Firstly, consider a similar liner system but at a higher applied stress ($\cong$ 100 kPa) that results in a decreased thickness, D = 0.007 m, but also a lower hydraulic conductivity, $k = 6\times10^{-11}$ m/s based on Rowe (1998). All other parameters are the same as previously assumed. The results, presented in terms of the factor F, are also shown in Fig. 3 and it can be seen that they are the same as for the previous case.

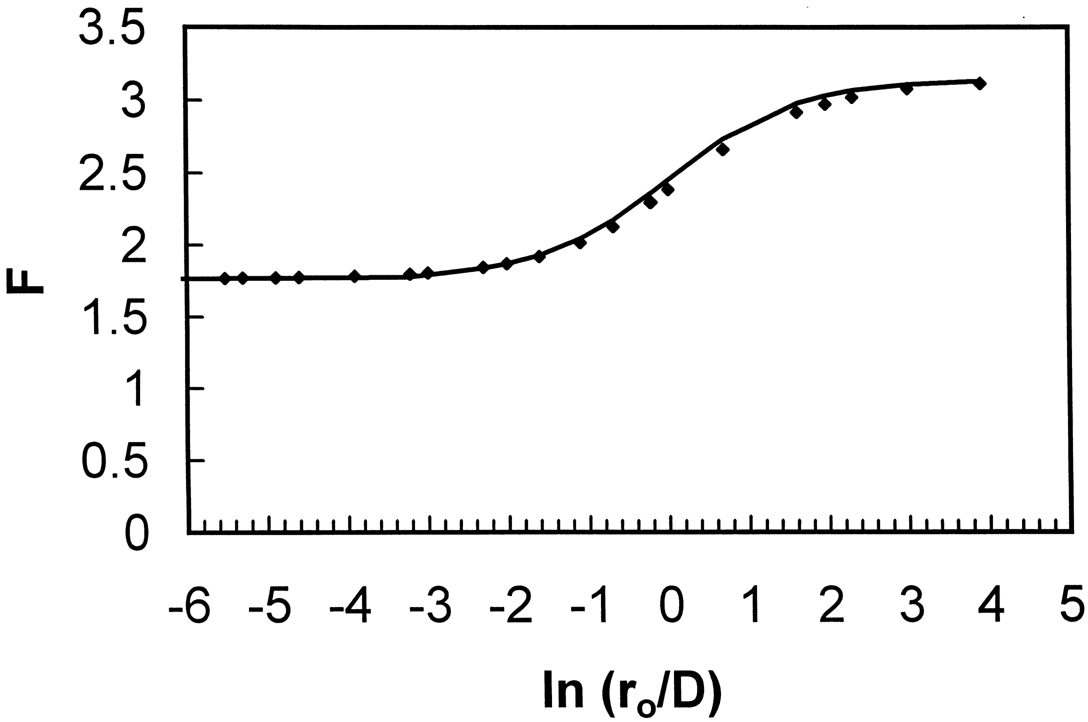

Figure 3. Variation in thickness factor F with ℓn of ratio of hole radius r_o and layer thickness D. Data points were calculated from rigorous analysis. Solid line is a plot of Eq. 34.

8 APPLICATION TO A COMPOSITE LINER INVOLVING A CCL

The theory presented in this paper is applicable to a full range of layer thicknesses D and head losses h_d. To illustrate its application to a composite liner system involving a geomembrane over compacted clay, consider a geomembrane over a D=0.75 m thick compacted clay liner (CCL) with $k=10^{-9}$ m/s as specified for both the primary liner in single liner systems and for both the primary and secondary liner in double liner systems by MoE (1998). It is again assumed that $h_w=0.3$ m (a typical design value) and $h_d=D+h_w$. This latter condition may be met where there is an active secondary leachate collection system (leak detection system) below a primary liner. It may also be met for either a single composite liner system or the secondary liner in a double composite liner system when the potentiometric surface is at the bottom of the compacted clay liner. It was found that for a given value of r_o/D, the dimensionless flow M was identical to that obtained for the GCL systems described in the previous section (see Fig. 2). Likewise, the fit to the approximate curve for F (Eq. 34) was the same (Figure 3), illustrating the applicability of this simple solution (to within 3%, which is adequate for most engineering applications). Of course, the theory and method of analysis presented in this paper can be used when more accurate results are required.

9 EFFECT OF ANISOTROPY

Based on Eqs. 23 and 31, it can be inferred that for $r_o/D \rightarrow 0$ any anisotropy in the clay liner will increase flow in proportion to $\sqrt{\frac{k_h}{k_v}}$ but that for $r_o/D \rightarrow \infty$ the effect would be negligible (k_v controls). The proposed theory allows one to analyze the effect of any potential anisotropy. To illustrate the interaction between r_o/D and the effect of $\sqrt{\frac{k_h}{k_v}}$, a series of analyses was initially performed for $k_h/k_v=10$. Figure 4 shows a plot of the ratio of the dimensionless flow calculated for the anisotropic case (M_a) divided by that for the isotropic case (M). As expected, the ratio approaches $\sqrt{10} = 3.16$ for small r_o/D and unity for large r_o/D. Based on these results, and results for other values of $\sqrt{\frac{k_h}{k_v}}$, it was found that the ratio M_a/M could be approximated by the function

$$\frac{M_a}{M} = 1 + f\sqrt{\frac{k_h}{k_v}} \qquad (36a)$$

where

$$f = 0.5\left(1 - \sqrt{\frac{k_v}{k_h}}\right)\left[1 - \tanh < \gamma + \delta \ell n(r_o / D) >\right] \qquad (36b)$$

and the parameters γ and δ are fitting parameters obtained to minimize the root mean square error to the approximation to the actual calculated value of M_a/M obtained using the theory presented in this paper. It was found that over the practical range $1 \leq k_h/k_v \leq 100$, an excellent fit was obtained for

$$\delta = 0.6 \qquad (36c)$$

$$\gamma = -0.167 - 0.0073\sqrt{\frac{k_h}{k_v}} \qquad (36d)$$

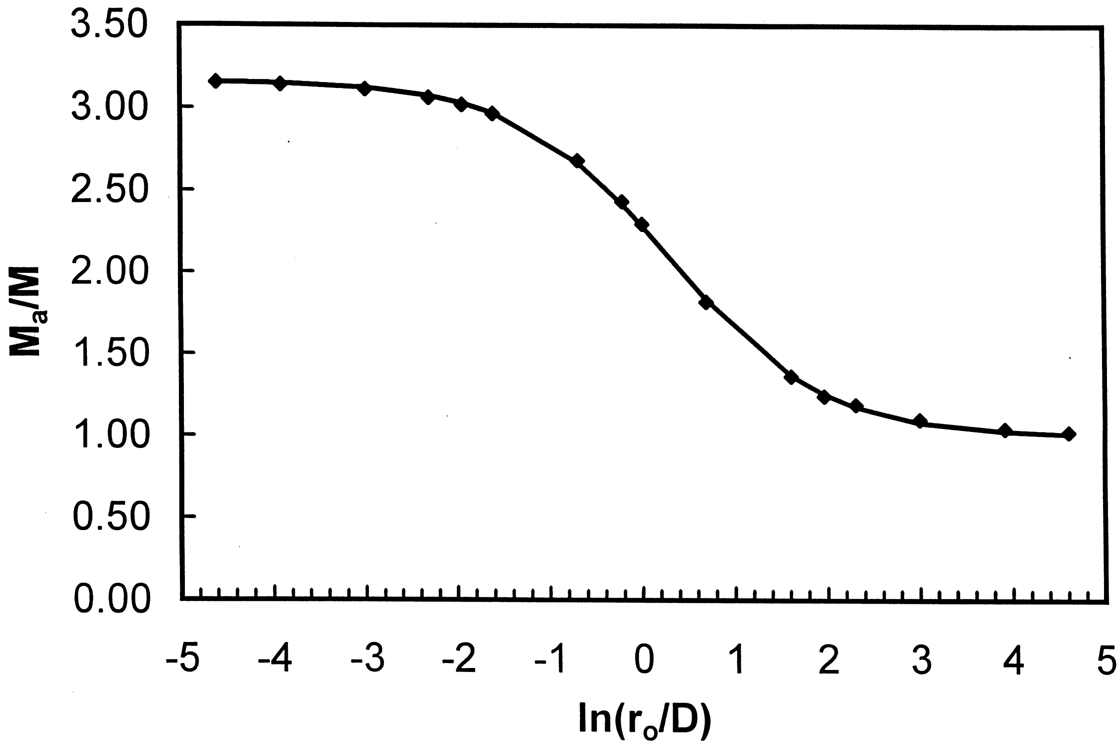

Figure 4. Ratio of dimensionless flow for an anisotropic soil (M_a) to that for an isotropic soil (M) against ℓn of ratio of hole radius r_o to liner thickness D for k_h/k_v=10. Data points are results from rigorous analysis. Solid line is a plot of Eq. 36.

10 CONCLUSION

A new semi-analytic solution to the problem of leakage through a hole in a geomembrane and the underlying clay has been presented. This solution covers the full range of layer thickness between a very thin and very thick layer (relative to hole size). The solution also allows consideration of anisotropy in the clay.

By using the proposed theory, it has been shown that the leakage, Q, through a circular hole of diameter r_o and an underlying isotropic clay layer with thickness D and hydraulic conductivity k_v can be approximated by a simple equation

$$\frac{Q}{r_o h_d k_v} = M = (4 + F\frac{r_o}{D}) \tag{33}$$

where

$$F = 2.455 + 0.685 \tanh < 0.6(\ell n r_o / D) > \tag{35}$$

and h_d is the head loss across the system.

For anisotropic soils with horizontal hydraulic conductivity k_h, it was found that the dimensionless flow, M_a, through the hole and soil could be related to the dimensionless flow, M, through an isotropic soil with $k=k_v$ by the simple expression:

$$\frac{M_a}{M} = 1 + f\sqrt{\frac{k_h}{k_v}} \quad (36a)$$

where

$$f = 0.5\left(1 - \sqrt{\frac{k_v}{k_h}}\right)\left[1 - \tanh < \gamma + 0.6\ell n(r_o / D) >\right] \quad (36b)$$

and

$$\gamma = -0.167 - 0.0073\sqrt{\frac{k_h}{k_v}} \quad (36d)$$

While Eqs. 33, 35 and 36 were obtained by curve fitting to the results of the more rigorous analysis, they have the advantage that they can be readily implemented in hand calculations as part of the design processes of composite landfill liners.

11 ACKNOWLEDGEMENTS

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APPENDIX

The general solution for transient conditions must satisfy

$$k_h\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)+k_v\frac{\partial^2 h}{\partial z^2}=m_v\frac{\partial h}{\partial t} \quad \text{(A1)}$$

and taking the Laplace and Fourier Transforms, this reduces to

$$\frac{\partial^2 \bar{H}}{\partial z^2}=\mu^2\bar{H} \quad \text{(A2)}$$

where $\mu^2=\dfrac{sm_v+\rho^2 k_h}{k_v}$

and $\rho^2=\alpha^2+\beta^2$

Equation A1 has a solution

$$\bar{H}=A\cosh\mu(h_a+z)+B\sinh\mu(h_a+z) \quad \text{(A3)}$$

and considering a layer of depth D with $\bar{H}=0 \;@\; z=-h_a$ gives

$$\bar{H}=\frac{\bar{V}_o\sinh\mu(h_a+z)}{k_v\mu\cosh\mu D}=\bar{V}_o\bar{L} \quad \text{(A4)}$$

and at the top of the layer, $z = D-h_a$,

$$\bar{H}_o=\bar{V}_o\frac{\sinh\mu D}{k_v\mu\cosh\mu D}=\bar{V}_o\frac{\tanh\mu D}{k_v\mu}=\bar{L}_o\bar{V}_o \quad \text{(A5)}$$

that, for steady-state conditions, reduces to

$$L_o = \frac{\tanh\left[\rho D\sqrt{\frac{k_h}{k_v}}\right]}{\rho\sqrt{k_h k_v}} \tag{A6}$$

and for a very deep layer this reduces to

$$L_o = \frac{1}{\rho\sqrt{k_h k_v}} \tag{A7}$$