Modelling impacts due to multiple landfill cells and clogging of leachate collection systems

R. Kerry Rowe and John R. Booker

Abstract: A semianalytic (finite layer) technique that will readily allow modelling of the interaction between different landfill cells is presented. The cells may be either adjacent or separated and may have different dimensions (length, thickness) and different source leachate characteristics. The technique also allows modelling of the finite service life of engineered components of the barrier system such as the service life of the primary leachate collection system and, if present, geomembrane liner. The application of the technique is illustrated by first considering the longitudinal expansion along the alignment of leachate migration of a landfill and examining the interaction between the original and subsequent landfill in terms of impact on the underlying aquifer. Consideration is then given to the vertical expansion of existing landfills and finally to the modelling of the finite service life of a primary leachate collection system and geomembrane liner. It is demonstrated that a vertical expansion has considerable potential for increasing long-term impact and that the impact is also controlled by the service life of the engineered system.

Key words: landfill, expansion, analysis, service life.

Résumé : L'on présente une technique semi-analytique (couche finie) qui permet aisément de modéliser l'interaction entre différentes cellules d'un enfouissement sanitaire. Les cellules peuvent être adjacentes ou séparées, et peuvent avoir différentes dimensions (longueur, épaisseur) et différentes caractéristiques du lessiviant d'origine. La technique permet également la modélisation de la vie utile limitée des composantes construites pour le système de barrière étanche, telle que la vie utile du système de collecte primaire des lessiviants et de la géomembrane d'étanchéité lorsque présente. L'application de la technique est illustrée en considérant en premier lieu l'expansion longitudinale le long de l'alignement de la migration du lessiviant d'un enfouissement, et en examinant l'interaction entre l'enfouissement original et subséquent en termes d'impact sur la nappe aquifère sous-jacente. L'on considère alors l'expansion verticale des enfouissements existants, et finalement la modélisation de la vie utile limitée d'un système de collecte primaire de lessivant et d'une géomembrane d'étanchéité. Il est démontré que l'expansion verticale a un potentiel considérable pour accroître l'impact à long terme et que l'impact est aussi contrôlé par la vie utile du système mis en place.

Mots clés : enfouissements sanitaires, expansions, analyse, vies utiles.
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Introduction

Due to the social and political problems associated with identifying new "greenfield" landfill sites, there is a growing trend to expand existing landfills. These expansions may be vertical and (or) horizontal or may involve the construction of separate but adjacent landfills. The incentive for expanding existing landfill sites is greatest when the existing landfill has not caused any significant impact on surface or groundwater during its period of operation. However, the fact that an existing landfill has not caused an environmental problem cannot be taken to mean that an expanded landfill will necessarily have negligible additional impact.

The potential impact of contaminant migration from a proposed landfill expansion will depend, in part, on how the older and newer cells are aligned relative to the direction of groundwater flow. If the expansion is perpendicular to the direction of groundwater flow, then from the perspective of contaminant transport modelling, it may be possible to consider the cells as isolated landfills (e.g., as discussed by Rowe and Booker 1985; Rowe et al. 1995*c*), since there will likely be negligible interaction provided that the construction of the new cell does not significantly change groundwater flow. Thus from this perspective it may be preferable to expand a landfill laterally (i.e., perpendicular to the direction of groundwater flow) rather than longitudinally (i.e., in the direction of ground flow). However, this may not always be possible due to nonhydrogeologic considerations. This paper focuses on the case where a longitudinal expansion is being considered and an assessment of the potential impact of the expansion on groundwater quality is required.

To assess the potential impact of a proposed expansion it is prudent to perform computer modelling, since, for a hydrogeologically suitable site, it is likely to take decades or even centuries for the full effects of the landfill on groundwater to become evident. Typically, the modelling needs to examine the development of an expanded landfill site which may take place over several decades. Therefore, modelling results cannot be taken as definitive but rather as an indication of trends that require careful consideration.

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For expanded landfills, the leachate in the older part of the landfill site may be decreasing in strength while that in the newer part of the site may be increasing in strength. If these cells are longitudinally aligned, it would be unrealistic to ignore the potential interaction between the two landfills in terms of impact on the aquifer system and equally unrealistic to assume both landfills were constructed simultaneously at peak leachate strength. Thus in these cases it is necessary to be able to model the interaction between adjacent landfills, allowing for different leachate generation histories.

For vertically expanded sites, the potential impact will be directly related to the increased mass of contaminant per unit area which will increase the contaminating lifespan of the landfill (i.e., the period of time during which the landfill is generating leachate which, if it escaped, could have an unacceptable impact on ground or surface waters; see Rowe et al. 1995*c*). The potential impact will also depend on the service life of the leachate collection system, and in modelling the potential impact it is important to consider the effects of leachate mounding due to clogging of the collection system (see Brune et al. 1991 and Rowe et al. 1995*b*, 1995*c*, and 1997*a* for a discussion of clogging of leachate collection systems).

An objective of this paper is to present a semianalytic (finite layer) theoretical formulation that allows the modelling of the interaction between multiple horizontally aligned landfill cells with different leachate time histories and to illustrate the application of the theory by examining a number of hypothetical examples. The second objective is to illustrate how the same theory can be used to examine the effect of a vertically expanded landfill upon potential impact in an underlying aquifer while giving consideration to the service life of a leachate collection system.

Theoretical formulation

The analyses reported in this paper are based on an extension to the two-dimensional finite layer formulation presented by Rowe and Booker (1985, 1991). The basic assumptions are that (*i*) the deposit can be subdivided into a number of layers which may each have different properties but where the properties in any one layer are the same at each point within the layer, (*ii*) contaminant transport is due to advection and (or) diffusion (i.e., density-driven transport is not considered), (*iii*) the horizontal and vertical groundwater velocities (v_x, v_z) in each layer are known at any time of interest, and (*iv*) contaminant transport can be idealized as being two dimensional. Thus for each layer we seek a solution to the two-dimensional advection–dispersion–diffusion–reaction equation

$$[1] \quad nD_{xx}\frac{\partial^2 c}{\partial x^2} + nD_{zz}\frac{\partial^2 c}{\partial z^2} - nv_x\frac{\partial c}{\partial x} - nv_z\frac{\partial c}{\partial z} = (n + \rho K_d)\frac{\partial c}{\partial t} + n\lambda c$$

where

D_{xx} and D_{zz} are the coefficients of hydrodynamic dispersion in the x and z directions;
v_x and v_z are the groundwater velocities in the x and z directions;
n is the porosity;
ρ is the dry density;
K_d is the partitioning–distribution coefficient;
λ is the first-order decay coefficient; and
c is the concentration at position (x, z) at time t in the particular layer, k, being considered and t denotes the time which has elapsed from the initiation of the stage being considered, subject to conditions of continuity of concentration and flux at the layer boundaries and subject to appropriate boundary and initial conditions as discussed below.

The earlier (Rowe and Booker 1985, 1991) formulation also assumed that (*v*) there was only one landfill of some fixed length, L, parallel to the direction of groundwater flow, and (*vi*) the landfill source concentration, c^*, was either constant for all time or decreased from some initial value, c_o, at a rate controlled by the initial mass of contaminant per unit area ($p\rho_w H_w$) and percolation through the landfill cover, q_c, based on consideration of conservation of mass; viz.

$$[2a] \quad c^*(t) = c_o - \frac{1}{H_r}\int_o^t q_c\, c^*(\tau)\mathrm{d}\tau - \frac{1}{LH_r}\int_o^t \int_{-L/2}^{L/2} f_T(x,\tau)\,\mathrm{d}x\,\mathrm{d}\tau \qquad \text{for} \quad -L/2 \le x \le L/2$$

$$[2b] \quad c^*(t) = 0 \qquad \text{for} \quad |x| > L/2$$

where

c_o is the source concentration at $t = 0$;
$f_T(x, \tau)$ is the mass flux into the barrier system at position x and time τ;
q_c is the volume of leachate collected per unit area of landfill;
$c^*(\tau)$ is the concentration in the landfill at time τ;
H_r is the reference height of leachate and represents the leachable mass of contaminant and can be given by $H_r = (p\rho_w H_w)/c_o$;
p is the proportion of total mass of waste represented by the leachable mass of the contaminant of interest;
ρ_w is the average density of the waste; and
H_w is the average thickness of the waste.

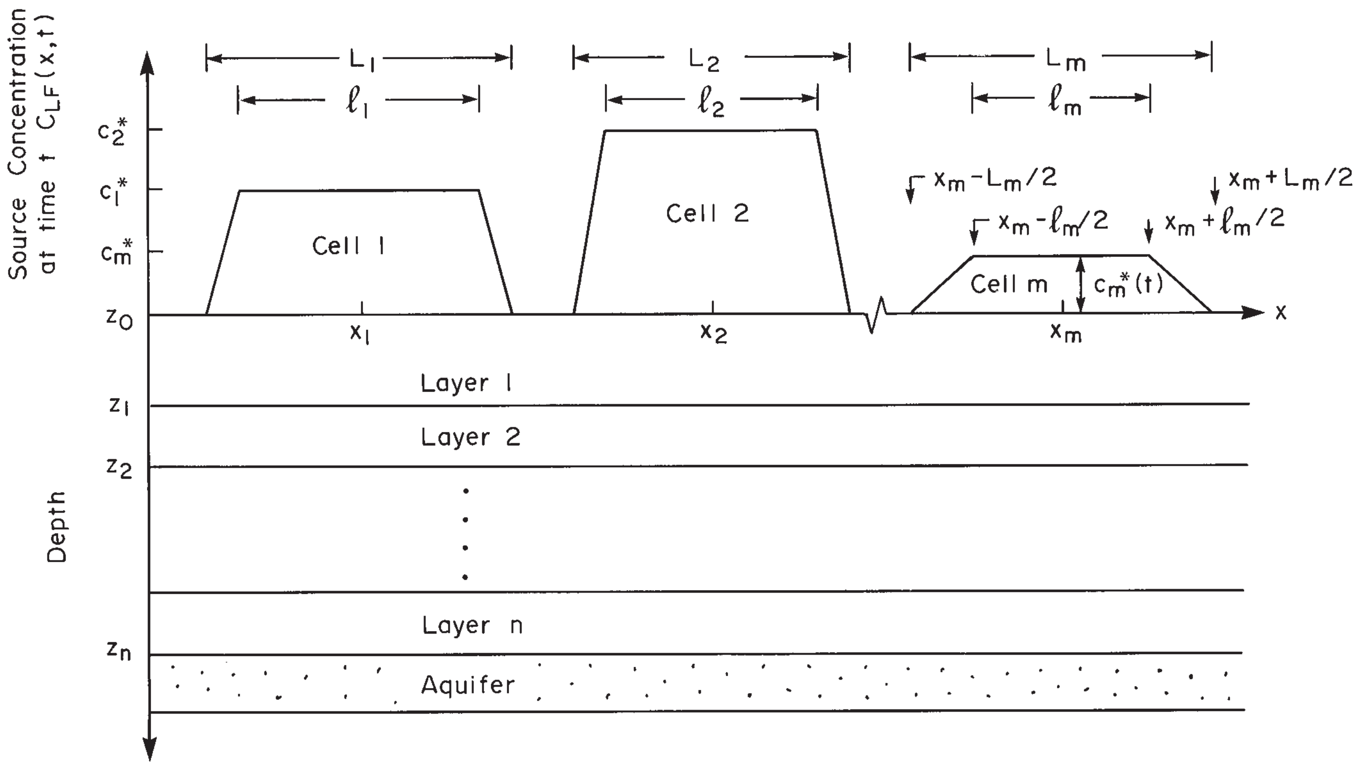
The present formulation relaxes these assumptions and permits consideration of multiple landfills and a concentration history that can vary in a more general way with time as summarized below and described in detail in Appendix A.

Consider a series of landfill cells as shown in Fig. 1 where it is assumed that the concentration acting on the barrier system for any cell m has a trapezoidal distribution, varying linearly from zero at the edge of the landfill cell ($x_m - L_m/2$) to an average value c_m^* across the base of the landfill cell, which is taken to be a distance ℓ_m in the direction parallel to groundwater flow and then back to zero at ($x_m + L_m/2$) where the length of the landfill between the perimeter ditches just outside the landfill cell is taken to be L_m for cell m. Any two cells m–1 and m may be contiguous (i.e., $x_{m-1} + L_{m-1}/2 = x_m - L_m/2$) or separated by some distance $\Delta x = (x_m - L_m/2) - (x_{m-1} + L_{m-1}/2)$, where $(x_m - L_m/2) > (x_{m-1} + L_{m-1}/2)$.

The landfill cells may be developed at different times. Thus, suppose that landfill cell 1 is developed first followed by the development of cells 2, 3,..., m. For cell 1 the concentration, c_{LF}, acting on the barrier system that separates the waste from some underlying aquifer can be approximated by

$$[3a] \quad c_{LF} = 0 \qquad x < x_1 - L_1/2$$

Fig. 1. Concentration loading on barrier system for multiple landfill cells at some time t.



[3b] $$c_{LF} = \frac{2c_1^*(x - x_1 + L_1/2)}{L_1 - \ell_1} \qquad x_1 - L_1/2 \le x < x_1 - \ell_1/2$$

[3c] $$c_{LF} = c_1^* \qquad x_1 - \ell_1/2 \le x \le x_1 + \ell_1/2$$

[3d] $$c_{LF} = \frac{2c_1^*(x_1 + L_1/2 - x)}{L_1 - \ell_1} \qquad x_1 + \ell_1/2 < x < x_1 + L_1/2$$

[3e] $$c_{LF} = 0 \qquad x_1 + L_1/2 \le x$$

For subsequent cells, the concentration loading on the barrier system due to cell m is given by

[4a] $$c_{LF} = 0 \qquad x_{m-1} + L_{m-1}/2 \le x < x_m - L_m/2$$

[4b] $$c_{LF} = \frac{2c_m^*(x - x_m + L_m/2)}{L_m - \ell_m} \qquad x_m - L_m/2 \le x < x_m - \ell_m/2$$

[4c] $$c_{LF} = c_m^* \qquad x_m - \ell_m/2 \le x \le x_m + \ell_m/2$$

[4d] $$c_{LF} = \frac{2c_m^*(x_m + L_m/2 - x)}{L_m - \ell_m} \qquad x_m + \ell_m/2 < x < x_m + L_m/2$$

[4e] $$c_{LF} = 0 \qquad x_m + L_m/2 \le x < x_{m+1} - L_{m+1}/2$$

except for the last cell where for $m = p$, [4e] is modified to

[4f] $$c_{LF} = 0 \qquad x_p + L_p/2 \le x$$

and for each value of $m = 1, 2, ..., p$, the value of c_m^* is given by the generalization of [2], viz.

[5] $$c_m^*(t) = c_{om} - \int_0^t \lambda_m c_m^*(\tau)\,d\tau - \frac{1}{H_{rm}}\int_0^t q_{cm} c_m^*(\tau)\,d\tau - \frac{1}{L_{avm}H_{rm}}\int_0^t \int_{x_m - L_m}^{x_m + L_m} f_T(x,\tau)\,dx\,d\tau$$

where λ_m is the first-order decay constant; $L_{avm} = (\ell_m + L_m)/2$; and the definitions of c_{om}, q_{cm}, $c_m^*(\tau)$, and H_{rm} parallel those defined earlier (c_o, q_c, $c^*(t)$, and H_r) but apply to cell m; and where the special case of a constant source concentration, $c_m^*(t) = c_{om}$, is given by [5] for $H_{rm} \to \infty$, $\lambda_m = 0$.

Vertical landfill expansions will be expected to result in a change in the source leachate concentration loadings but can be modelled for cell m by changing the parameters c_{om}, H_{rm}, and, if appropriate, q_{cm} as a function of time. If a leachate collection system fails, then the groundwater velocity through the barrier system and the underlying aquitards and aquifers can be expected to change with time as a leachate mound develops. Thus to implement these features in a finite-layer analysis it is necessary to be able to (i) starting at $t_o = 0$ ($m = 1$), proceed with an analysis from $t = t_{m-1}$ to some time $t = t_m$ with a given set of parameters; (ii) store the concentration profile at time $t = t_m$; and (iii) restart the analysis using the initial concentration profile at $t = t_m$ and the new set of parameters for the period t_m to t_{m+1}.

It should be noted that this "time marching" of the solution is common in finite-element formulations for contaminant transport analysis; however, in finite-element analyses the time marching is a numerical necessity and the choice of time increment $\Delta t = (t_m - t_{m-1})$ may affect the results even if there is no change in properties (i.e., source concentration history, groundwater velocities, etc.). However, with the finite-layer approach developed in Appendix A, the time marching is only related to changes in physical conditions.

In the basic finite-layer formulation, the number of layers to be modelled can correspond to the number of different physical layers with different hydrogeologic properties. Thus there is no discretization error. However, for the present case the need to store the concentration profile at each time corresponding to a change in physical properties necessitates some interpolation of concentrations between layer boundaries as described in Appendix A. Thus the accuracy may, to some extent, depend on the number of sublayers into which each physical layer is divided, with an increasing number of sublayers reducing the level of numerical approximation as described by Rowe and Booker (1995).

Like many other computer models, the approach adopted in this paper models the aquifer and aquitards as layers where the hydrogeologic properties may vary from one layer to another but are assumed to be constant within any given layer. Careful consideration should be given to the potential effects of variability and complexity (both vertically and horizontally) of the natural geology and hydrostratigraphy when performing impact assessments. This may require sensitivity analysis using the proposed model or, for highly variable cases, the use of alternative models if there is sufficient data to define the variability of the deposit.

Application of theory

A number of hypothetical examples will be discussed in the following subsections. The findings from these analyses should not be generalized or applied to actual sites, since a change in parameters could change the findings. These examples are intended to illustrate how the theory might be used. In any analysis of an existing or proposed facility one would need to use parameters relevant to that particular situation and perform a wider range of sensitivity studies (to examine the effect of uncertainty in hydrogeologic parameters) than is possible in this paper.

In these examples, only chloride will be examined. Chloride is a relatively mobile contaminant that generally experiences negligible retardation or degradation ($K_d = 0$, $\lambda = \infty$). Although the drinking-water objective (250 mg/L) for chloride is based on aesthetic considerations, it can often control the design of landfills in jurisdictions (like Ontario) where an increase in chloride is limited by government guidelines (e.g., Ontario Ministry of the Environment and Energy 1994). In actual design situations, other contaminants (e.g., organic contaminants) would also be examined. In modelling these contaminants, sorption (e.g., see Rowe et al. 1995*c*) and biodegradation (e.g., see Fetter 1993; Rowe 1995; Rowe and Weaver 1998; Rowe et al. 1997*b*) can be modelled by appropriate choice of parameters (e.g., K_d and λ in eq. [1]). Sorption and biodegradation will both serve to reduce the potential impact; however, each individual case must be evaluated separately to assess whether the potential impact is acceptable.

The rate of concentration decreases with time (which is governed by the ratio q_c/H_r for chloride and by $(\lambda + q_c/H_r)$ for organics). The model allows all source parameters λ, q_c, H_r, and c_o to be varied at each time t_m for each landfill cell, and hence the effects of a change in cover performance (q_c), a change in source concentration (c_o), a change in mass of contaminant (H_r), and the rate of biodegradation (λ) can be examined as appropriate for a given situation. For simplicity of presentation, these parameters will not be changed (unless otherwise noted) in the following.

These examples examine the common situation where the aquitard is very thin relative to the landfill dimensions and consequently the effect of any horizontal flow in the aquitard is likely to be negligibly small provided that the horizontal hydraulic conductivity of the aquitard is low. However, the theory can readily model a horizontal flow in one or more of the aquitard layers if that were considered appropriate for the hydrogeologic conditions being modelled at a particular site.

In the following examples, the coefficient of hydrodynamic dispersion in the aquitard will be taken to be isotropic ($D_{xx} = D_{zz}$) and equal to the diffusion coefficient. This is likely appropriate for the relatively low groundwater velocities examined (see Rowe 1987; Rowe et al. 1995*c*). However, mechanical dispersion and different values of D_{xx} and D_{zz} could be modelled where appropriate. In these examples, it was assumed that the contaminant was uniformly mixed in the aquifer over the thickness considered (3–4 m) but that the concentration in the aquifer varied with position x as dictated by a mass-balance equation for the aquifer as described in detail by Rowe and Booker (1985, 1995). This is convenient but not a necessary assumption; variation in concentration with depth in the aquifer can also readily be modelled (see Rowe et al. 1985, 1991).

The horizontal dispersivity, α_H, in the aquifer was assumed to be zero for the cases considered. Other dispersivities could be modelled and in an actual application it would be normal to do a sensitivity analysis to examine the effect of uncertainty regarding α_H. Although α_H is generally regarded to be scale dependent, it is unlikely that this parameter will be known in the type of problem being considered here and it would be appropriate to adopt whatever reasonable value gave the most conservative predicted impact. The diffusion coefficent for chloride in the aquifer was taken to be 0.03 m^2/a based on Rowe and Badv (1996).

For simplicity of presentation, in the following analyses it is assumed that the initial background concentration of chloride in the clay and aquifer is zero and the reported chloride concentration represents an increase in concentration relative to this background value.

Longitudinal expansion of landfill

Consider the situation shown schematically in Fig. 2. Landfill No. 1 extending a distance of 300 m in the direction of groundwater flow is constructed such that the source chloride concentration increases to a maximum of 1500 mg/L over a period of 15 years and then decreases with time after closure. The landfill is assumed to have 15 m of waste (including daily and intermediate cover) with an average apparent density of 600 kg/m^3 and with chloride representing 2 g/kg of the total waste mass. At year 30, a second landfill (landfill No. 2) is developed downgradient of landfill No. 1 to an average thickness of 25 m over a period of 25 years, with the maximum (average) concentration being 2500 mg/L and the waste

Fig. 2. Spatial and temporal distribution of source concentration with time for cases 1 and 2.

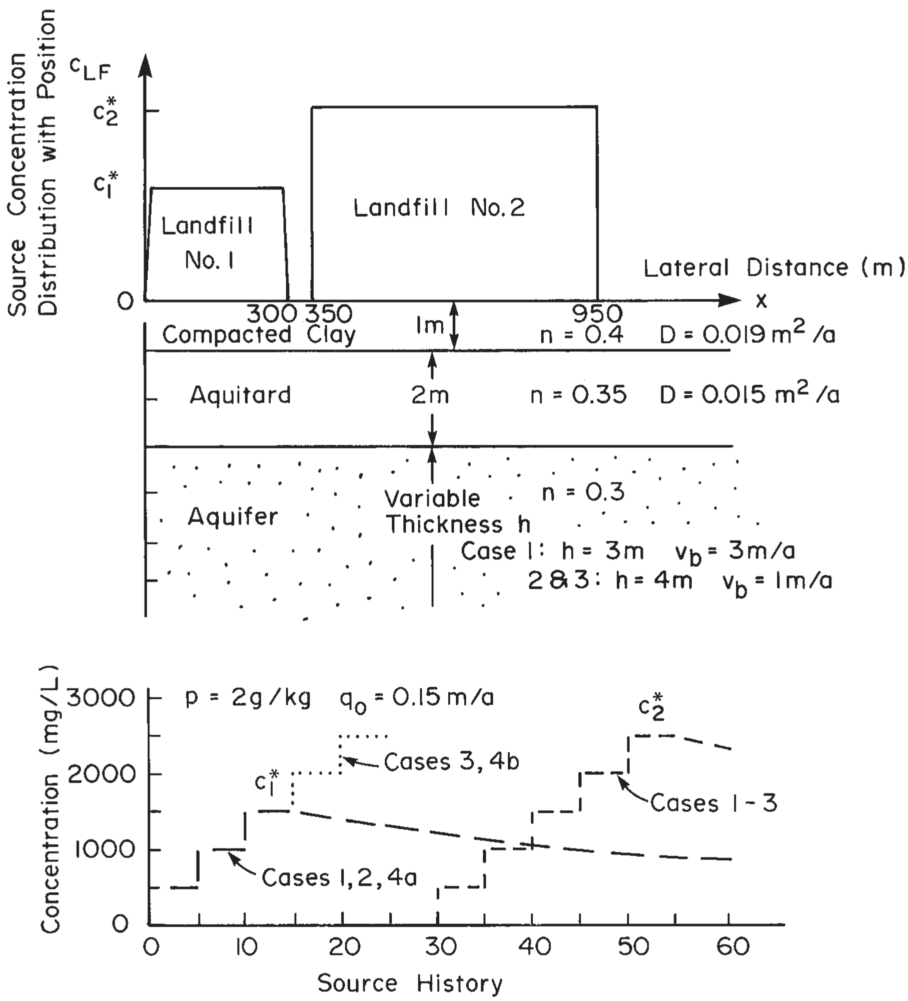
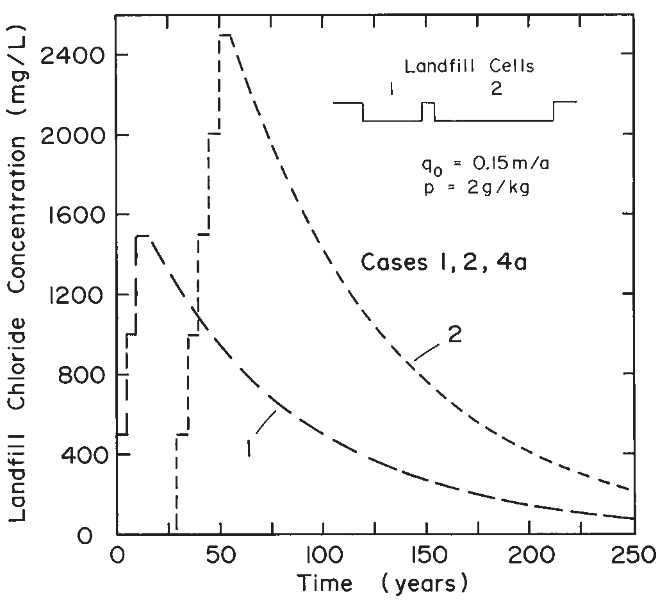


Fig. 3. Variation in landfill source concentration with time for cases 1, 2, and 4*a*.



density and mass proportions for chloride being the same as those for landfill No. 1. The barrier between the waste and underlying aquifer consisted of compacted clay over a natural aquitard with properties as given in Fig. 2. For this example, it is assumed that the potentiometric surface in the aquifer is 3.3 m above the top of the aquifer and that there is a leachate collection system which, while operating, controls the leachate head to 0.3 m above the aquitard. Thus there is no vertical flow in the aquitard (i.e., pure diffusion) and the leachate collection system collected $q_c =$ 0.15 m/a under normal operating conditions for both landfills.

Figure 3 shows the variation in chloride concentration with time in the landfill and if the contaminating lifespan is defined as the period of time required for the concentration of chloride to drop to a value below the typical drinking-water objective of 250 mg/L, then from the time of initial landfilling ($t = 0$) it would be about 160 and 240 years, respectively, before cells 1 and 2 reach their contaminating lifespan.

Consider case 1 with an aquifer 3 m thick with a horizontal Darcy velocity v_b (also known in the groundwater literature as the Darcy flux, specific discharge, volumetric flux density, or discharge velocity) of 3 $(m^3/a)/m^2 \equiv 3$ m/a (and corresponding to an average linearized groundwater velocity of 10 m/a). Figure 4 shows the calculated variation in concentration with

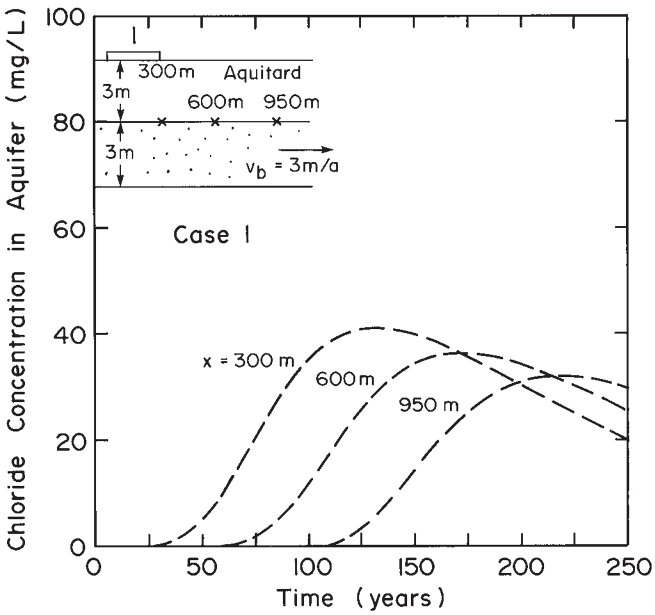


Fig. 4. Calculated impact at x = 300, 600, and 950 m due to cell 1 alone: case 1.

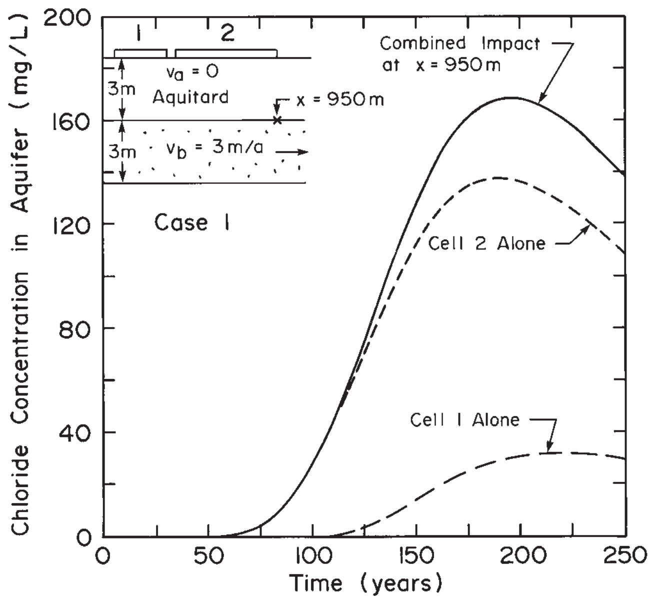


Fig. 5. Calculated impact at x = 950 m (below downgradient edge of landfill cell 2): case 1.

time at x = 300 m (downgradient edge of cell 1), 600 m, and 950 m, assuming that cell 2 was never constructed. For this case, the peak concentration at the edge of cell 1 (x = 300 m) of about 41 mg/L is reached at about 130 years, and at x = 950 m the peak impact due to landfill cell 1 is 32 mg/L at 220 years. Thus the model indicates that there would be some attenuation (~22%) between 300 and 950 m simply due to diffusion from the aquifer (which has a relatively high concentration) back up into the aquitard (which has a relatively low concentration) downgradient of cell 1 (remembering that the horizontal dispersivity was assumed to be zero in these examples). The calculated impact would be acceptable in most regulatory systems if, as assumed here, there was initially a low chloride concentration in the aquifer. Figure 5 shows the variation in calculated concentration with time at x = 950 m for cell 1 alone, cell 2 alone, and cells 1 and 2 together. If one simply superimposed the peak impacts at 950 m due to cells 1 (32 mg/L) and 2 (138 mg/L) alone, one obtains a peak impact of about 170 mg/L at 190 years compared with a value of 168 mg/L obtained when the two cells are analyzed together. Thus in this case the impact of the two landfills could be estimated by a superposition of the peak impacts of the individual landfills. However, this is not necessarily the case for other combinations of parameters as illustrated below.

For case 2, consider a 4 m thick aquifer with a horizontal Darcy velocity v_b (Darcy flux) of 1 $(m^3/a)/m^2 \equiv 1$ m/a (corresponding to a groundwater velocity of 3.3 m/a). Figure 6 shows the calculated variation in concentration with time at x = 300, 600, and 950 m. First, it will be noted that there is an increase in attenuation from 22% for case 1 to 37% for case 2 with distance between 300 and 950 m. This is due to greater diffusion of chloride from the aquifer back into the aquitard for x > 300 m as a result of the greater concentration gradient. The concentrations are larger but not in direct proportion to the change in flow in the aquifer. Most interesting, however, is the combined effect of cells 1 and 2 as can be appreciated by comparing the calculated concentrations of x = 950 m shown in Fig. 7. Cell 1 gives a peak impact at x = 950 m of approximately 50 mg/L at about 420 years. Cell 2 alone gives a peak impact of about 211 mg/L at 280 years, and the combined impact of cells 1 and 2 gives a peak impact of 220 mg/L at 300 years. Thus the combined effect of the two cells is only marginally greater than that of cell 2 alone, and the effect of cell 1 is relatively small. The impact of the two cells would be even more independent at a lower horizontal velocity.

The foregoing two cases are close to (but not at) the two possible extremes of complete superposition of the peak impacts and totally independent impact. It is also noted that the peak impact ranges between about 170 and 220 mg/L for a relatively small range of variability of aquifer thickness (3–4 m) and Darcy velocity (Darcy flux) v_b in the aquifer (3 m/a to 1 m/a) (i.e., the total flow changing by a factor of 2.25) which are well within the typical ranges of uncertainties regarding the hydrogeologic parameters. This illustrates the need to perform sensitivity studies but also illustrates the fact that simple superposition of peak impacts at 950 m (approximately 260 mg/L) for analyses of isolated cells would give a misleading assessment of the potential impact (220 mg/L). Addition of the concentrations at a given time due to two independent cells gives a smaller but variable level of error which depends on the hydrogeologic parameters and hence is difficult to establish without doing the combined analysis.

To highlight the points made above, case 3 is similar to case 2 except that cell 1 is assumed to be constructed over 25 years with a peak waste thickness of 25 m and peak concentration of 2500 mg/L (i.e., the same as that for cell 2). Figure 8 shows the variation in source concentration with time for the case, and the contaminating lifespan with respect to chloride reaching 250 mg/L for cell 1 is increased from 160 to 210 years (cell 2 remains at 240 years). Figure 9 shows the variation in calculated concentrations at x = 950 m for cell 1 alone, cell 2 alone, and cells 1 and 2 combined. The results for cell 2 alone are

Fig. 6. Calculated impact at x = 300, 600, and 950 m due to cell 1 alone: case 2.

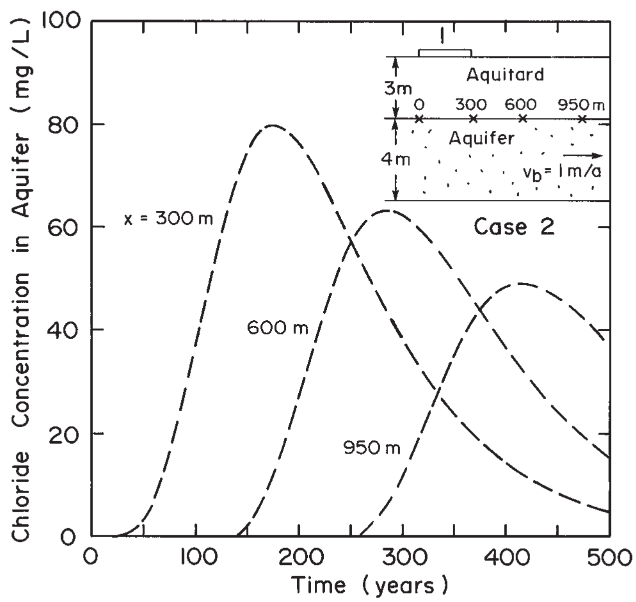


Fig. 7. Calculated impact at x = 950 m (below downgradient edge of cell 2): case 2.

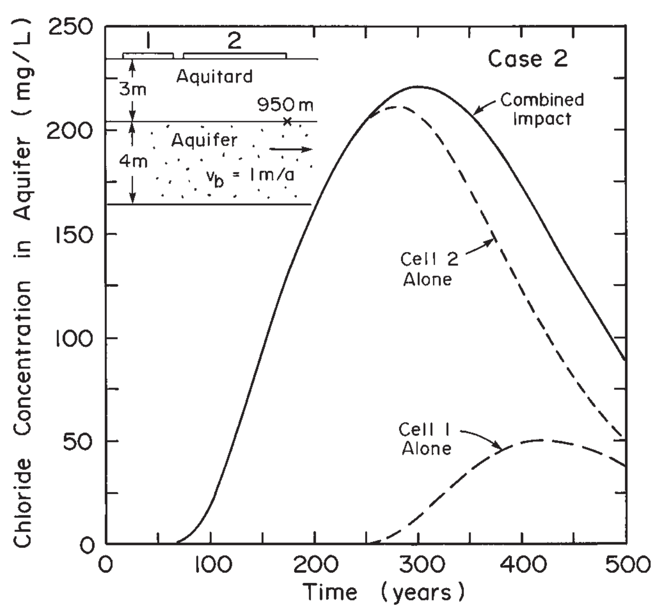


Fig. 8. Variation in landfill source concentration with time: cases 3 and 4b.

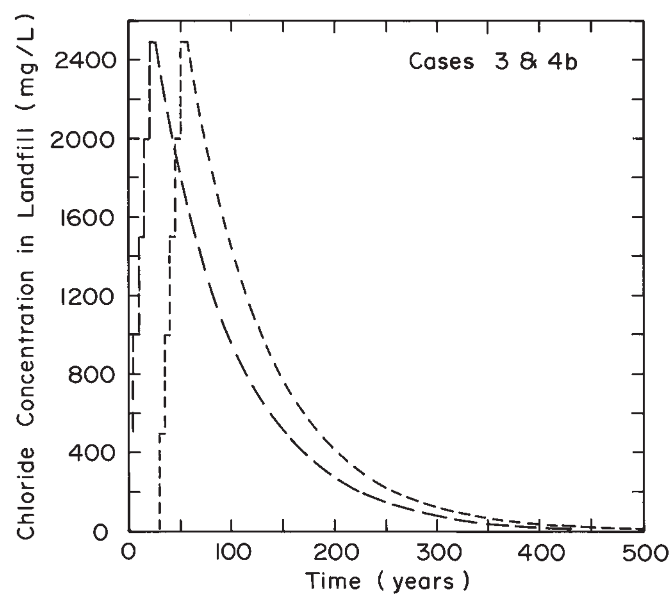
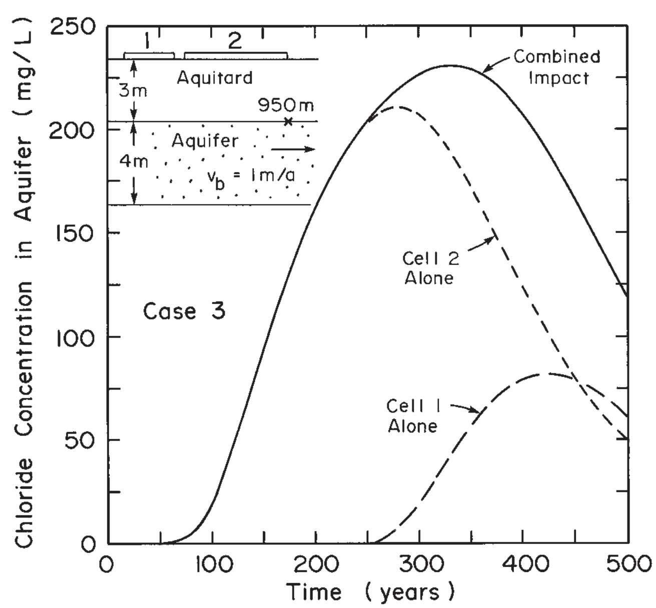


Fig. 9. Calculated impact at x = 950 m (below downgradient edge of cell 2): case 3.

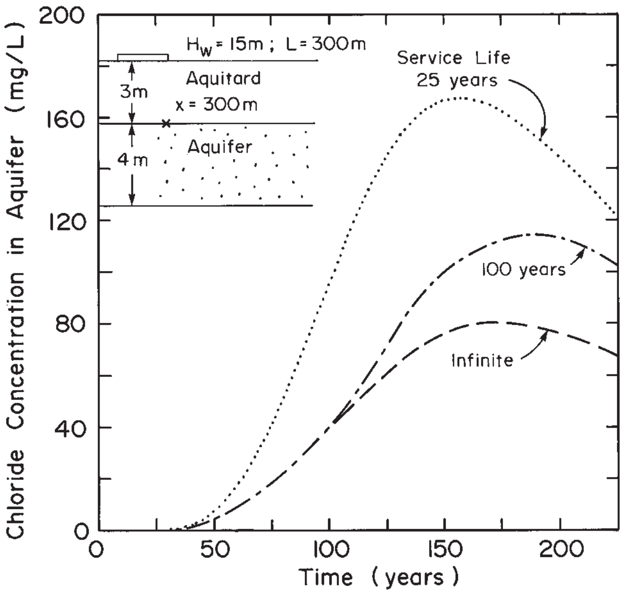


exactly the same as those for case 2, since this cell was not changed. The concentrations for cell 1 are increased in direct proportion to the change in peak concentration (i.e., by the ratio 2500/1500 = 1.667, since the concentration was assumed to be proportional to the thickness of waste). However, the interaction between the two cells is such that one cannot simply scale the results for cell 1 and superimpose them on those for cell 2 to get the combined peak impact; superposition of concentration does not hold. Here, the error would also be relatively modest (about 6 mg/L) but unconservative.

In this case, the peak impact of multiple cells at a given point can be estimated by superimposing the calculated concentration for each of the different cells at that point at corresponding times; however, this superposition does not always hold and the approach may underestimate the peak impact. Thus it is desirable to check the effect of interaction on the peak impact.

It is also worth noting that in each case there was no readily detectable impact on the aquifer until after about 40 years and hence at the time landfill cell 2 is commenced (30 years) there would be no direct evidence that cell 1 would have any impact. This is due to the time it takes for contaminant to diffuse through the system. However, it would be quite wrong to interpret this to mean that there would be no subsequent impact.

Fig. 10. Effect of finite service life of leachate collection system on calculated impact for 90 000 t/ha landfill: case 4*a*.



It is also worth noting that 30 years after closure of cell 2 (i.e., at 85 years), the impact at $x = 950$ m would only be about 10 mg/L (or less) and hence quite small for all three cases, whereas the peak impacts range from about 170 mg/L (case 1) to about 230 mg/L (case 3). This implies that there is very little interaction between the two cells. The level of interaction would be even less for a smaller horizontal flow in the aquifer. This situation arises because the time lag between contaminant entering the aquifer below cell 1 and its migration to below cell 2 is increased.

In the event that these impacts were not acceptable, one could also model the impacts for values of x greater than 950 m to assess the size of the attenuation zone that might be required to meet regulatory requirements at the site boundary.

Vertical expansion of landfill

When considering the vertical expansion of a landfill the two potentially most significant considerations with respect to contaminant impact are the extension of the contaminating lifespan and the effect of a finite service life of the engineered system (e.g., the leachate collection system). To illustrate this, consider the hydrostratigraphy considered in cases 2 and 3 with the potentiometric surface 0.3 m above the level of the compacted clay liner and hence zero vertical flow under design operating conditions. Also consider a vertical expansion of cell 1 from 15 to 25 m and consider the impact at the edge of the landfill (i.e., $x = 300$ m).

As discussed in a previous section, the time required for the source concentration to decrease to 250 mg/L is increased from 160 to 210 years (assuming that the expansion from 15 to 25 m occurred without any delay, the peak chloride concentration is proportional to the landfill height and the additional waste contained the same mass of leachable chloride). Assuming an indefinite service life of the leachate collection system, the peak impact increases from approximately 80 mg/L at about 170 years for the 15 m high landfill to approximately 132 mg/L at 180 years for the 25 m high expanded landfill (compare infinite landfill service life curves in Figs. 10 and 11).

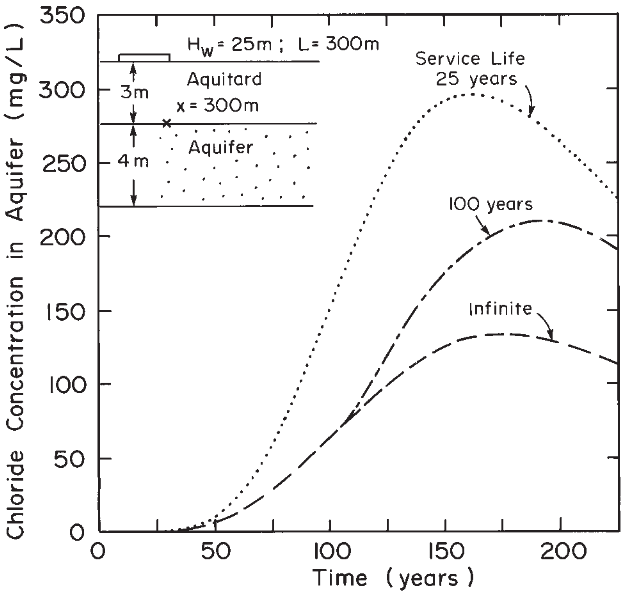
The foregoing calculations assume an indefinite service life of the primary leachate collection system. However, the service life of leachate collection systems is not indefinite. A leachate collection system may be regarded as having clogged when it can no longer maintain the leachate level at the design level. This invariably means that the hydraulic conductivity has dropped many orders of magnitude but the "clogged" collection layer will still be far more permeable than the underlying liner and hence will not generally provide any significant hydraulic resistance to vertical leachate migration. There is limited field evidence that clogging may also reduce the conductivity of a clay "liner" (see Rowe et al. 1995*c*); however, the decrease, if it exists, is of no practical significance.

The finite service life of leachate collection systems raises the question as to what would be the impact if the service life of the leachate collection system was estimated to be 25 years (poor design) or 100 years (good design) as discussed by Rowe et al. (1994). After failure, it is assumed that a leachate mound builds up over a period of 5 years to an average height of 3 m where the mound is controlled using purge wells in the waste (the feasibility of doing this must be carefully considered but is beyond the scope of this paper; see Rowe and Nadarajah 1996). Assuming a consolidated hydraulic conductivity of the compacted clay liner of 5×10^{-11} m/s and a hydraulic conductivity of the natural aquitard of 1×10^{-9} m/s, the downward Darcy velocity (Darcy flux) under leachate mounding of 3 m is $v_a = 0.0043$ m/a ((m^3/a)/m^2). This is still a very small downward Darcy velocity but, as will become evident, it can have a significant effect on calculated impact. The groundwater velocity in the aquifer is assumed to increase from 3.3 to 4.3 m/a, corresponding to an increase in Darcy velocity (Darcy flux) from 1 to 1.32 (m^3/a)/m^2 to satisfy continuity of flow.

Figure 10 shows the calculated impact at the downgradient edge of the landfill for case 4, where the average waste thickness was taken to be 15 m (90 000 t/ha). Results are shown for the service life of the primary leachate collection system of 25 and 100 years as well as the indefinite base case. With a long service life the peak impact was about 80 mg/L (at 170 years). Reducing the service life to 100 years gave a peak impact of about 115 mg/L at 190 years which may still be acceptable in Ontario (where the maximum is 125 mg/L for zero background concentration) and is well below the drinking-water objective of 250 mg/L. With a service life of 25 years, the 15 m thick landfill gave a peak impact of about 165 mg/L at 160 years and exceeds the allowable impact in Ontario but is still below the drinking-water objective. Increasing the size of the landfill to 25 m thick (150 000 t/ha) increases the effect of the service life of the leachate collection system, as can be appreciated by comparing Figs. 10 and 11. For the large landfill even an infinite service life of the collection system gives an impact that exceeds that permitted in Ontario but is well below the drinking-water objective of 250 mg/L. For the larger landfill a 25 year service life gives an impact of almost 300 mg/L (at 160 years), which well exceeds the drinking-water objective.

These results show how the method of analysis can be used to model failure of components of the engineered system and highlight the importance of considering the effect of increased landfill size on groundwater quality when proposing a vertical

Fig. 11. Effect of finite service life of leachate collection system on calculated impact for 150 000 t/ha landfill: case 4*b*.



expansion. In this case, the 90 000 t/ha landfill would likely have an acceptable impact provided it included a well-designed leachate collection system; an expansion of the landfill to 150 000 t/ha would have an unacceptable impact at the edge of the landfill based on Ontario regulations (Ontario Ministry of the Environment and Energy 1994). Thus the analysis indicates that engineering modifications must be considered if one was wishing to expand this landfill. Alternatively, the model could be used to establish the attenuation zone required downgradient of the landfill to meet regulatory requirements at the site boundary. A suitable attenuation zone could then be acquired to allow expansion to the desired height. Another alternative may be to expand the landfill laterally (perpendicular to the direction of groundwater flow) rather than vertically.

Liner system involving a geomembrane (case 5)

As a final example of the application of the theory presented in this paper, consider the case of a United States Environmental Protection Agency “Subtitle D” (United States Office of the Federal Register 1994) barrier system consisting of a high density polyethylene (HDPE) geomembrane over 0.6 m of compacted clay liner which, in this case, is constructed over a 2.4 m thick natural aquitard (i.e., the total barrier thickness is as previously discussed for cases 1–4). Also in this case the hydraulic conductivity of the compacted clay is taken to be 1×10^{-9} m/s and the potentiometric surface in the aquifer is at the level of the top of the liner. Thus with an operating leachate collection system and geomembrane installed with high quality control and quality assurance (one small hole per hectare) there is no advection and contaminant transport is by diffusion. The diffusion coefficient for chloride in the clayey soil and geomembrane is taken to be 0.018 and 1×10^{-7} m^2/a, respectively (based on Rowe et al. 1995*a*, 1996).

The failure of geomembranes requires some discussion. If well-constructed geomembranes provide an excellent barrier to the movement of fluid and to the diffusion of certain constituents in leachate such as chloride and volatile fatty acids (see Rowe et al. 1996), and provided the geomembrane remains intact, the leakage and diffusion of these contaminants through a composite liner system will be small. On the other hand, some organic compounds can readily diffuse through geomembrane, and these contaminants may control impact in some circumstances (see Rowe et al. 1995*b*, 1996).

High density polyethylene geomembrane will degrade with time. The rate of degradation will depend on temperature and the presence of free radicals that encourage chain scission. The degradation of a geomembrane can be slowed by the introduction of antioxidant packages; however, the antioxidant will deplete with time due to a number of processes, which include diffusion of the antioxidant out of the geomembrane. The greatest risk to geomembrane performance is environmental stress cracking, which would allow leachate to get below the geomembrane and thence migrate through the clay by advection and diffusion. As demonstrated by Rowe et al. (1998), with the formation of cracks, the geomembrane ceases to be effective as an advective diffusion barrier. Based on available evidence (see Koch et al. 1988; Koerner et al. 1990; Rowe et al. 1994) the service life of geomembrane at 25°C is estimated to be about 150 years (Ontario Ministry of the Environment and Energy 1996). Here it is assumed that cracking begins at this time and that the performance degrades over the next few decades until the geomembrane has no effect on controlling either advective or diffusive transport.

The impacts are calculated for a 300 m wide landfill with two different average thicknesses of waste (i.e., 7.5 and 25 m, corresponding to 45 000 and 150 000 t/ha and peak source concentration c_0 of 1000 and 2500 mg/L, respectively; $p = 2$ g/kg in both cases). Assuming that the leachate collection system fails at 100 years (Rowe et al. 1994; Ontario Ministry of the Environment and Energy 1996), a leachate mound will build up to a height of 7.5 m controlled by landfill geometry for the smaller (7.5 m) landfill and an average height of 12 m controlled by mounding between the perimeter drains (spacing $\ell = 300$ m and the hydraulic conductivity of the waste $k_{(\mathrm{waste})} = 10^{-6}$ m/s; see Rowe et al. 1995*c*) for the larger landfill (where the perimeter drains are assumed to be 4 m above the base of the landfill). The mound is assumed to develop over a 15 and 20 year period for the 7.5 and 12 m mounds, respectively, and gives rise to advective transport through the geomembrane as shown in Table 1. Since it is assumed here that the geomembrane starts to fail at 150 years and that the failure occurs over a period of time, the Darcy velocity will increase to the peak values as shown in Table 1 for the conditions examined. For the case where there is a 12 m leachate mound, the increased advection through the barrier due to the assumed failure of the geomembrane can be expected to lower the leachate mound with time to 9 m, consequently resulting in a reduction in Darcy velocity (see Table 1).

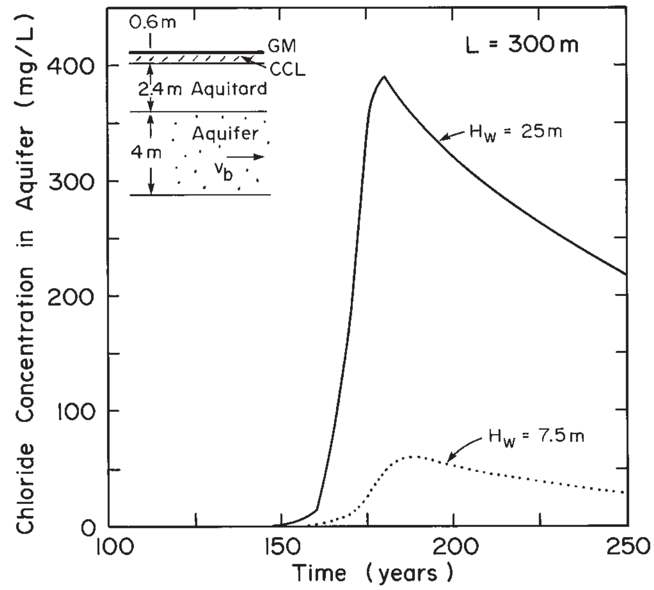
The calculated variation in concentration with time for these two landfill sizes is shown in Fig. 12. In all cases, the geomembrane provides a very effective barrier to contaminant migration during the period of its operation, and negligible quantities of chloride reached the aquifer. However, when the geomembrane fails, the leachate mound and hydraulic conductivity of the clayey barrier (10^{-9} m/s) are such that the contaminant quickly migrates to the aquifer and the peak impacts occur shortly after failure of the geomembrane. For the smaller landfill, the peak impact is only about 60 mg/L and hence would

Table 1. Leachate mound and Darcy velocities used in case 5.

Time period (years)	Leachate mound (m)	Darcy velocity (m/a)
	$H_w = 7.5$ m	
0–100	~0	0
100–107.5	2.5	0.000833
107.5–115	5.0	0.00167
115–150	7.5	0.0025
150–160	7.5	0.028
160–170	7.5	0.0535
>170	7.5	0.079
	$H_w = 25$ m	
0–100	0	0
100–110	4	0.0013
110–120	8	0.0027
120–150	12	0.004
150–160	12	0.042
160–170	12	0.084
170–180	12	0.126
180–190	10.5	0.11
>190	9	0.095

Note: Darcy velocity is also known as Darcy flux or specific discharge, with units of $(m^3/a)/m^2 \equiv m/a$.

Fig. 12. Calculated chloride impact in aquifer at downgradient edge of landfill considering 100 year service life of leachate collection system and 150 year service life of geomembrane liner. CCL, compacted clay liner; GM, geomembrane.



likely meet the requirements of the Ontario Ministry of the Environment and Energy (1994). However, for the assumed conditions, the larger landfill gives a peak impact of about 390 mg/L and hence greatly exceeds the typical drinking-water objective of 250 mg/L. This illustrates how the size of the landfill must be considered when assessing the potential impact of a landfill and also illustrates the fact that a given liner system may be adequate for one landfill size but may not be adequate for a larger landfill; this can only be assessed by modelling.

Conclusions

A semianalytic (finite layer) technique has been presented which will readily allow modelling of the interaction between different landfill cells that may be either adjacent or separated and may have different dimensions (length, thickness) and different source leachate characteristics. The technique also allows modelling of the finite service life of engineered components of the barrier system such as the service life of the primary leachate collection system and, if present, geomembrane liner.

The application of the technique has been illustrated by first considering the longitudinal expansion of a landfill (in the direction of groundwater flow) and examining the interaction between the original and subsequent landfill in terms of impact on the underlying aquifer. It is shown that the level of interaction can be highly dependent on the aquifer properties and that for even a small variation in these properties (well within the typical range of hydrogeologic uncertainty) the interaction can range from small to significant. This demonstrates the need to both examine potential interaction and perform sensitivity studies over a reasonable range of hydrogeologic uncertainty. For the range of cases considered, it is found that a reasonable estimate of impact would have been obtained by superimposing the impacts (at the same position and time) of the individual landfill cells; however, this finding should not be generalized without further investigation.

Some general conclusions that can be drawn from the cases examined in the paper are as follows:

(1) Proper assessment is required when considering longitudinal and (or) vertical expansion of existing landfill facilities. Each case would require independent examination using site-specific parameters.

(2) The fact that an existing landfill is currently performing adequately and is not presently creating an adverse impact does not necessarily mean that it will not cause an adverse impact at some future time or that the landfill can be safely expanded. Modelling is required to interpret current monitoring results and predict future impacts of both the current and proposed expanded landfill.

(3) The analysis shows that under certain hydrogeologic conditions it may not be possible to expand a site vertically or longitudinally without engineering modification to mitigate an otherwise unacceptable future impact.

(4) For landfills constructed over thin deposits of clay (either natural aquitards or engineered liners), there may be cases where a single liner system is not sufficient to allow a vertical landfill expansion when due consideration is given to the service life of the primary leachate collection system and, if present, the geomembrane liner.

(5) The service life of the primary leachate collection system may be the key parameter controlling contaminant impact for landfills designed with single clay (either natural or engineered) liners. The model readily allows consideration of changed conditions when the service life of the leachate collection system is reached.

(6) The service life of the geomembrane may be the key parameter controlling the impact of chloride on an underlying

aquifer for landfills with a well-constructed composite (geomembrane over clay) liner.

(7) Although not explicitly illustrated by the examples, the model allows consideration of the migration of organic compounds including diffusion through a geomembrane, sorption, and biodegradation at different rates in the leachate, clay, and aquifer. The interested reader is referred to Rowe et al. (1995*a*, 1996, 1997*a*, 1997*b*) for more information on these issues.

It should be emphasized that lateral, longitudinal, or vertical expansion of an existing landfill facility may represent quite viable solutions to a particular waste disposal problem. This paper simply cautions that each case should be carefully examined and provides a technique that may be useful for this examination in some cases.

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Appendix A

The transport of contaminant in soils is essentially an evolutionary phenomenon. This means that provided the properties of the soil are known and the boundary conditions affecting the transport of contaminant in the soil are specified, then it is possible to predict the transport of contaminant in the soil until these two conditions change, provided of course that the initial concentration of the contaminant is known.

This simple observation enables the analysis of quite complex landfill operations to be broken up into the analysis of relatively simple stages. Thus, for example, the first stage may consist of the construction of a landfill cell; the second stage, which may commence several years later, may involve modelling of the construction of an adjacent cell. Subsequently, there may also be a change in the groundwater regime which needs to be modelled.

Appendix A will consider the analysis of one such stage of the modelling of a site that consists of a number of landfill cells constructed on a horizontally stratified soil which consists of a number of homogeneous layers, which in turn overlay a transmissive aquifer, as illustrated in Fig. 1. There are n soil layers and the layers are assumed to be bounded by the $(n + 1)$ horizontal planes $z = z_0, z = z_1, ..., z = z_n$. At the particular stage of development the landfill consists of m cells.

Behaviour in the individual layers

It will be assumed that the length (in the y direction) of the landfills perpendicular to the direction of groundwater flow is relatively large and so the variation in that direction can be ignored.

The two-dimensional advection–dispersion–diffusion–reaction equation for any layer is given by [1]. We seek to solve this equation for each layer, invoking continuity of concentration and flux at the layer boundaries, subject to boundary conditions which will be discussed later and the initial condition

$$\text{[A1]} \quad c = c_0\,(x, z) \quad \text{when } t = 0$$

where c_0 is the known distribution of concentration of contaminant at the commencement of the current stage.

Equation [1] can be simplified by introducing a Laplace transform with respect to time and a Fourier transform with respect to x. These are defined as follows:

$$\text{[A2}a\text{]} \quad \bar{c}\;(x,z,s) = \int_0^\infty c(x,z,t)\mathrm{e}^{-st}\,\mathrm{d}t$$

$$\text{[A2}b\text{]} \quad C(\xi,z,t) = \int_{-\infty}^{+\infty} c(x,z,t)\mathrm{e}^{-ix\xi}\,\mathrm{d}x$$

where i is $\sqrt{-1}$, and s and ξ are transform integration parameters. The equations of mass transport then become

$$\text{[A3]} \quad nD_{zz}\frac{\partial^2\bar{C}}{\partial z^2} - nv_z\frac{\partial\bar{C}}{\partial z} = n(S\bar{C} - \Phi)$$

where

$$S = D_{xx}\xi^2 + i\xi v_x + \lambda + Rs$$

$$\Phi = RC_0 = R\int_{-\infty}^{+\infty} c_0(x,z)\mathrm{e}^{i\xi x}\mathrm{d}x \qquad \text{and}$$

$$R = 1 + \frac{\rho K_\mathrm{d}}{n}$$

Now to be specific, consider the kth layer ($z_j < z < z_k$) where $j = k–1$ and $k = 1,..., n$. By appropriate selection of layer thickness, it is usually possible to approximate the quantity Φ with sufficient accuracy in the form

$$\Phi = E\mathrm{e}^{\varepsilon z}$$

where ε and E are given by

$$\text{[A4]} \quad \begin{aligned} \varepsilon &= \frac{\ln(\Phi_k) - \ln(\Phi_j)}{z_k - z_j} \\ \ln(E) &= -\frac{z_j\ln(\Phi_k) - z_k\ln(\Phi_j)}{z_k - z_j} \end{aligned}$$

and

$$\Phi_j = \Phi(z_j)$$

$$\Phi_k = \Phi(z_k)$$

and the numerical accuracy of the approximation can be checked by repeating the analysis using more layers of lesser thickness of the sublayers used to simulate the deposit. Equation [A3] then has the solution

$$\text{[A5]} \quad \bar{C} \;=\; \frac{\Phi}{G} + a\mathrm{e}^{\alpha z} + b\mathrm{e}^{\beta z}$$

where

$$G = -D_{zz}(\varepsilon - \alpha)(\varepsilon - \beta)$$

$$\text{[A6]} \quad \begin{aligned} \alpha &= -\frac{v_z}{2D_{zz}} + \left(\frac{v_z^2}{4D_{zz}^2} + \frac{S}{D_{zz}}\right)^{1/2} \\ \beta &= -\frac{v_z}{2D_{zz}} - \left(\frac{v_z^2}{4D_{zz}^2} + \frac{S}{D_{zz}}\right)^{1/2} \end{aligned}$$

and a and b are constants to be determined. Consequently, it is possible to establish the following relationship between concentration and vertical flux for this layer:

$$\text{[A7]} \quad \begin{bmatrix} \bar{F}_j \\ -\bar{F}_k \end{bmatrix} = -\begin{bmatrix} A_k \\ B_k \end{bmatrix} + \begin{bmatrix} Q_k & R_k \\ S_k & T_k \end{bmatrix}\begin{bmatrix} \bar{C}_j \\ \bar{C}_k \end{bmatrix}$$

where

$\bar{C}_{j,k}$ are the transformed concentrations at the node planes $z = z_{j,k}$

$\bar{F}_{j,k}$ are the transformed values of the vertical flux at the node planes $z = z_{j,k}$

$$A_k = [n\,D_{zz}(\varepsilon - \alpha - \beta)\Phi_{k-1} + Q_k\,\Phi_{k-1} + R_k\,\Phi_k]/G$$

$$B_k = [\,-n\,D_{zz}(\varepsilon - \alpha - \beta)\Phi_k + S_k\Phi_{k-1} + T_k\Phi_k]/G$$

and Q_k, R_k, S_k, and T_k are as defined by

$$Q_k = nD_{zz}(\beta\mathrm{e}^{-\alpha H} - \alpha\mathrm{e}^{-\beta H})/(\mathrm{e}^{-\alpha H} - \mathrm{e}^{-\beta H})$$

$$R_k = nD_{zz}(\beta - \alpha)/(\mathrm{e}^{\alpha H} - \mathrm{e}^{\beta H})$$

$$S_k = -nD_{zz}(\beta - \alpha)/(\mathrm{e}^{-\alpha H} - \mathrm{e}^{-\beta H})$$

$$T_k = -nD_{zz}(\beta\mathrm{e}^{\alpha H} - \alpha\mathrm{e}^{\beta H})/(\mathrm{e}^{\alpha H} - \mathrm{e}^{\beta H}) \qquad \text{and}$$

$$H = z_k - z_{k-1}$$

Considerations of continuity of concentration and flux at the interface of adjacent boundaries lead to the relationship

$$\text{[A8]} \quad \begin{bmatrix} Q_1 & R_1 & 0 & \cdots & 0 & 0 \\ S_1 & Q_2{+}T_1 & R_2 & \cdots & 0 & 0 \\ 0 & S_2 & Q_3{+}T_2 & \cdots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & Q_n{+}T_{n-1} & R_n \\ 0 & 0 & 0 & \cdots & S_n & T_n \end{bmatrix}\begin{bmatrix} \bar{C}_0 \\ \bar{C}_1 \\ \bar{C}_0 \\ \vdots \\ \bar{C}_{n-1} \\ \bar{C}_n \end{bmatrix} = \begin{bmatrix} A_1{+}B_0 \\ A_2{+}B_1 \\ A_3{+}B_2 \\ \vdots \\ A_n{+}B_{n-1} \\ A_{n+1}{+}B_n \end{bmatrix}$$

where

$$B_0 = \bar{F}_0 \qquad \text{and}$$

$$A_{n+1} = -\bar{F}_{n+1}$$

Behaviour in the aquifer

It will be assumed that vertical variation of concentration in the aquifer is relatively small and for this case it has been shown (Rowe and Booker 1985) that the concentration within the aquifer satisfies

$$[\text{A9}a]\quad h_\text{b}\left(n_\text{b}D_\text{b}\frac{\partial^2 c_\text{b}}{\partial x^2} - v_\text{b}\frac{\partial c_\text{b}}{\partial x}\right) + f_\text{b} = n_\text{b}h_\text{b}\left(\frac{\partial c_\text{b}}{\partial t} + \lambda_\text{b}c_\text{b}\right)$$

where for the aquifer layer

h_b is the thickness;
n_b is the porosity;
D_b is the horizontal coefficient of hydrodynamic dispersion;
v_b is the horizontal Darcy velocity (Darcy flux, specific discharge);
f_b is the flux from the landfill into the aquifer at position x;
λ_b is the first-order decay coefficient; and
c_b is the concentration in the aquifer at position x and time t;

with the initial condition that

$$[\text{A9}b]\quad c_\text{b} = c_\text{b0} \qquad \text{when } t = 0$$

When a Laplace transform and a Fourier transform (eq. [A2]) are applied to [A9a], it is found that

$$[\text{A10}]\quad \bar{F}_\text{b} = \Omega_\text{b}\bar{C}_\text{b} - A_\text{b}$$

where

$$\Omega_\text{b} = hn_\text{b}\left(s + D_H\xi^2 + \frac{iv_\text{b}\xi}{n_\text{b}} + \lambda_\text{b}\right)$$

$$A_\text{b} = hn_\text{b}C_\text{b0}$$

and

$$C_\text{b0} = \int_{-\infty}^{+\infty} c_\text{b0}(x)\text{e}^{-\text{i}\xi x}\,\text{d}x$$

Interaction with the landfill

To complete the solution it is necessary to establish the relationship between the concentration in the various landfill cells and the concentration distribution in the soil profile. For the sake of simplicity, it will be assumed that the distribution of contaminant within each landfill can be approximated by the trapezoidal distribution shown in Fig. 1. An examination of the mass balance within the mth landfill cell leads to the relationship

$$[\text{A11}a]\quad c_m^*(t) = c_{0m}^* - \int_0^t \lambda_m c_m^*(\tau)\,\text{d}\tau - \frac{1}{H_{\text{r}m}}\int_0^t q_{\text{c}m}c_m^*(\tau)\,\text{d}\tau - \frac{1}{L_{\text{av}m}H_{\text{r}m}}\int_0^t\left[\int_{x_m-L_m/2}^{x_m+L_m/2} f_\text{T}(x,\tau)\text{d}x\right]\text{d}\tau$$

where $L_{\text{av}m} = 0.5(\ell_\text{m} + L_m)$

When [A11a] is transformed using a Laplace transform it becomes

$$[\text{A11}b]\quad (s + \mu_m)\bar{c}_m^*(s) = c_{0m}^* - \frac{1}{L_{\text{av}m}H_{\text{r}m}}\left[\int_{x_m-L_m/2}^{x_m+L_m/2} \bar{f}_\text{T}(x,s)\text{d}x\right]$$

where

$$\mu_m = \lambda_m + \frac{q_{\text{c}m}}{H_{\text{r}m}}$$

Incorporating [A10] into [A8], the relationship becomes

$$[\text{A12}]\quad \begin{bmatrix} Q_1 & R_1 & 0 & \cdots & 0 & 0 \\ S_1 & Q_2{+}T_1 & R_2 & \cdots & 0 & 0 \\ 0 & S_2 & Q_3{+}T_2 & \cdots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & Q_n{+}T_{n-1} & R_n \\ 0 & 0 & 0 & \cdots & S_n & T_n \end{bmatrix}\begin{bmatrix} \bar{C}_0 \\ \bar{C}_1 \\ \bar{C}_0 \\ \vdots \\ \bar{C}_{n-1} \\ \bar{C}_\text{b} \end{bmatrix} = \begin{bmatrix} A_1{+}\bar{F}_\text{T} \\ A_2{+}B_1 \\ A_3{+}B_2 \\ \vdots \\ A_n{+}B_{n-1} \\ A_{n+1}{+}B_n \end{bmatrix}$$

This equation can be used to establish the relationship between the surface concentration and the surface flux in the form

$$[\text{A13}]\quad \bar{F}_\text{T} = \Psi\bar{C}_0 - \Theta$$

where

$$\Psi = P_1 - Q_1R_1[\text{e}_1^\text{T}K^{-1}\text{e}_1] \qquad \text{and}$$

$$\Theta = A_1 - Q_1[\text{e}_1^\text{T}K^{-1}w]$$

with

$$\text{K} = \begin{bmatrix} Q_2{+}T_1 & R_2 & 0 & \cdots & 0 & 0 \\ S_2 & Q_3{+}T_2 & R_3 & \cdots & 0 & 0 \\ 0 & S_3 & Q_4{+}T_3 & \cdots & 0 & 0 \\ \vdots & & & \cdots & & \vdots \\ 0 & 0 & 0 & \cdots & Q_n{+}T_{n-1} & R_n \\ 0 & 0 & 0 & & S_n & T_n + \Omega_\text{b} \end{bmatrix}$$

$$\text{e}_1^\text{T} = [\;1 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 1\;]$$

$$w^\text{T} = [\;A_2{+}B_1 \quad A_3{+}B_2 \quad A_4{+}B_3 \quad \cdots \quad A_n{+}B_{n-1} \quad A_\text{b}{+}B_n]$$

If there is a total of p landfill cells, then

$$[\text{A14}]\quad \bar{C}_0 = \sum_1^p \bar{c}_m^*\,U_m$$

where

$$U_m = \frac{1}{c_m^*}\int_{x_m-L_m/2}^{x_m+L_m/2} \text{e}^{-\text{i}x\xi}\,\bar{c}_\text{LF}(x,s)\text{d}x$$

$$= \frac{4\text{e}^{-\text{i}\xi x_0}\left[\cos\left(\dfrac{\xi\ell_m}{2}\right) - \cos\left(\dfrac{\xi L_m}{2}\right)\right]}{(L_m - \ell_m)\xi^2}$$

Equation [A13] and the Fourier inversion theorem can be used to calculate the flux distribution at the base of the landfills, and it is found that

$$[\text{A15}]\quad \bar{f}^\text{T} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}[\Psi\bar{C}_0 - \Theta]\text{e}^{\text{i}x\xi}\,\text{d}\xi$$

It is now possible to determine the average concentration in each landfill using the mass-balance equation (eq. [A11*b*]), and it is found that

$$\text{[A16]}\quad (s+\mu_m)\bar{c}_m^*(s) = c_{0m}^* - \frac{1}{L_{\text{avm}}H_{\text{rm}}}\left[\int_{x_m-L_m/2}^{x_m+L_m/2} \bar{f}_{\text{T}}(x,s)\,\mathrm{d}x\right]$$

or, when all cells are considered,

$$\text{[A17]}\quad Z\bar{c}^* = d$$

where
Z is a square matrix of dimension m with elements z_{pq} given by

$$z_{pq} = (s+\mu_p)\delta_{pq} + \frac{1}{2\pi L_{\text{av}m}H_{\text{r}m}}\int_{-\infty}^{+\infty}\left[\Psi V_p U_q\right]\mathrm{e}^{\mathrm{i}x\xi}\mathrm{d}\xi \qquad \text{and}$$

d is a vector of dimension m with components d_p given by

$$d_p = c_{0p}^* + \frac{1}{2\psi L_{\text{av}m}H_{\text{r}p}}\int_{-\infty}^{+\infty}[\Theta V_p]\mathrm{e}^{\mathrm{i}x\xi}\mathrm{d}\xi$$

The quantity U_q was defined in [A14], and the quantity V_p is defined by

$$V_p = \int_{x_p-L_p/2}^{x_p+L_p/2} \mathrm{e}^{\mathrm{i}x\xi}\,\mathrm{d}x = \frac{2\mathrm{e}^{\mathrm{i}\xi x_0}\sin\left(\frac{\xi L_p}{2}\right)}{\xi}$$

Once the transformed average landfill concentrations $\bar{c}^*$ have been determined, the transformed surface concentration $\bar{C}_0$ can be determined from [A14], and this can be substituted into [A12] to determine the transformed layer concentrations. The values of these quantities in the physical plane can be obtained by using Talbot's (1979) algorithm to invert the Laplace transform and numerical integration to evaluate the Fourier transform. This completes the analysis of the particular stage being examined. Subsequent stages can be evaluated in a similar way.