Technical Paper by N. Touze-Foltz, R.K. Rowe, and N. Navarro

LIQUID FLOW THROUGH COMPOSITE LINERS DUE TO GEOMEMBRANE DEFECTS: NONUNIFORM HYDRAULIC TRANSMISSIVITY AT THE LINER INTERFACE

ABSTRACT: Rates of liquid flow through composite liners are intimately linked with the existence of a transmissive layer between the soil liner and the geomembrane. The aim of this paper is to propose analytical solutions for estimating the rate of liquid flow through composite liners for the case where two zones of different hydraulic transmissivities coexist in the transmissive layer. Solutions are given for both the axisymmetric and two-dimensional cases. The development of these analytical solutions is directly based on solutions that have been proposed previously for uniform hydraulic transmissivity. Results tend to show, for the simple cases evaluated, that, in situations where there is variable hydraulic transmissivity, neglecting the variability can result in a significant overestimation of the rate of liquid flow. This preliminary approach shows the importance of taking into account a realistic distribution of hydraulic transmissivity in attempts to estimate the rate of liquid flow through composite liners and highlights the need for more research in this area.

KEYWORDS: Geomembrane, Leakage, Defect, Variable hydraulic transmissivity.

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1 INTRODUCTION

Geomembranes used for the containment of landfills may have holes caused by inadequate seaming, puncture, tears, etc. A recent synthesis of studies involving electrical leak detection systems (Rollin and Jacquelin 2000) reports a hole density varying from 2 to 26 defects per hectare after installation of the geomembrane. These defects form preferential advective leachate flow paths through the geomembrane. A number of analytical solutions (Brown et al. 1987; Rowe 1998; Touze-Foltz et al. 1999) have been developed to quantify rates of liquid flow for holes in flat or wrinkled geomembranes, where the transmissive layer between the underlying soil and the geomembrane is of uniform thickness and, thus, where the hydraulic transmissivity is uniform.

In practice, it is likely that soil and geomembrane surfaces are not flat and parallel. Vallejo and Zhou (1995) and Dove and Frost (1996) have shown that geomembrane surfaces are not perfectly smooth and exhibit a certain roughness. Moreover, in the field, geomembranes expand when they are heated by the sun and wrinkles appear. This is one of the three sources of imperfections affecting contact conditions between the soil and the geomembrane as identified by Rowe (1998). The other two are related to the soil surface: (i) protrusions related to particle size distribution; and (ii) undulations/ruts that may appear following compaction. As a consequence, the transmissive layer between both elements of a composite liner may rarely be of uniform thickness under field conditions. Recent experiments seem to confirm this hypothesis (Navarro 1999; Touze-Foltz 1999). This nonuniformity between the geomembrane and the soil surface results in a variable hydraulic transmissivity of the transmissive layer.

The objective of the present paper is to extend the work of Touze-Foltz et al. (1999) and to propose simple analytical solutions for estimating rates of liquid flow through composite liners in the case of variable hydraulic transmissivity. Consideration is given to cases where two zones with different hydraulic transmissivity values coexist in the transmissive layer. In this simple configuration, analytical solutions can be obtained. Solutions are presented for the case of a circular hole and then for the case of a damaged wrinkle, for a range of boundary conditions, defined by Touze-Foltz et al. (1999). The solution to the problem of liquid flow through composite liners where there is variable hydraulic transmissivity can be obtained based on existing analytical solutions. The expressions of the hydraulic head developed by Touze-Foltz et al. (1999) for the case of uniform hydraulic transmissivity are used wherever the hydraulic transmissivity is uniform in the transmissive layer. It is recognised that the idealisations of the variable transmissivity adopted here are probably not representative of reality, since, in practical cases, the spatial distribution of hydraulic transmissivity values may be two-dimensional; however, they do serve to show potential importance of the influence of this spatial variability and provide motivation for additional study.

Section 2 of the present paper defines the assumptions regarding the geometry, boundary conditions, and hydraulics of the composite liners used to establish the new solutions. Expressions of the hydraulic head in the transmissive layer and of the rates of liquid flow obtained are presented in Section 3. The objective of Section 4 is to discuss the definition of an equivalent hydraulic transmissivity in the case where the hydraulic transmissivity is nonuniform in the transmissive layer. Section 5 illustrates the application of the solution to show the influence of the variability of the hydraulic transmissivity relative to the case of a uniform hydraulic transmissivity, with particular reference to the influence on the rate of liquid flow through composite liners.

2 ASSUMPTIONS AND LIMITATIONS

The solutions to be developed consider the problem of infiltration through a composite liner where there is either (i) a circular hole in a flat geomembrane, or (ii) a damaged wrinkle in a geomembrane. Assumptions common to both cases are presented in Section 2.1. Assumptions specific to the two cases are detailed in Sections 2.2 and 2.3, respectively, for the axi-symmetric (circular hole in flat surface) and two-dimensional (hole in a wrinkle) cases as defined by Touze-Foltz et al. (1999).

2.1 Common Assumptions

The basic problem definition (Figures 1 and 2) follows from Rowe (1998) and Touze-Foltz et al. (1999) and involves a geomembrane resting on a low-permeability clay liner of thickness H_L and hydraulic conductivity k_L. This low-permeability clay liner may be either a compacted clay liner (CCL) or a geosynthetic clay liner (GCL) and will be simply called a "soil liner". The *z*-axis origin corresponds to the top of the soil liner with upward being positive. The soil liner rests on a more permeable foundation or attenuation layer of thickness H_f and hydraulic conductivity k_f which itself rests on a highly permeable layer that can be either an aquifer or a secondary collection layer. Following from Brown et al. (1987) and Giroud and Bonaparte (1989), it is assumed

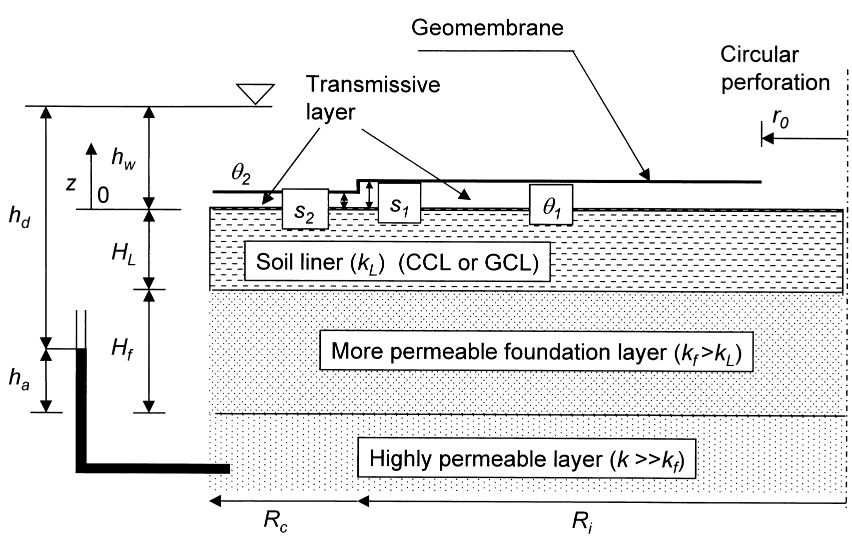

Figure 1. Schematic showing a hole of radius r_0, the two zones of different interface transmissivity, and the underlying strata (modified from Touze-Foltz et al. 1999).

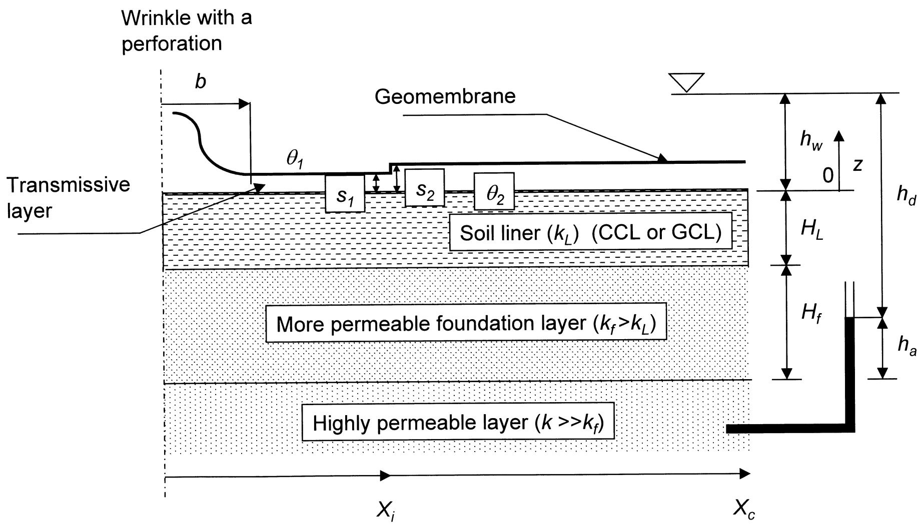

Figure 2. Schematic showing a wrinkle with a perforation in a geomembrane, the two zones of different interface transmissivity, and the underlying strata (modified from Touze-Foltz et al. 1999).

that the geomembrane is not in perfect contact with the soil liner surface and that there is a transmissive layer between the surfaces of the soil liner and the geomembrane. In contrast to all previous solutions, it is assumed in the present paper that there are two zones with different hydraulic transmissivity values: (i) the first one, with transmissivity θ_1, extends out from the centre of the hole or wrinkle to a distance R_i (or X_i); and (ii) the second, with transmissivity θ_2, extends beyond this point.

In the following, it is assumed that: (i) flow is under steady-state conditions; (ii) the soil liner and the foundation layer are saturated; (iii) flow through the soil liner and underlying foundation layer is vertical; and (iv) the head loss at the interface between the two zones with uniform hydraulic transmissivity values is negligible. Indeed the continuous quantity in the transmissive layer at this interface between both zones with different hydraulic transmissivity values is the hydraulic pressure, not the hydraulic head. The hypothesis of negligible head loss at the interface between both zones with different hydraulic transmissivity values will be discussed in Section 5.1.

Based on continuity of vertical liquid flow, the equivalent hydraulic conductivity, k_s, corresponding to the liner and the foundation layer is given by (Rowe 1998):

$$\frac{H_L + H_f}{k_s} = \frac{H_L}{k_L} + \frac{H_f}{k_f} \tag{1}$$

When a hydraulic head, h_w, is applied on top of the composite liner, the maximum mean hydraulic gradient, i_s, through the liner and foundation is given by:

$$i_s = 1 + \frac{h_w - h_a}{H_L + H_f} \tag{2}$$

where h_a is the hydraulic head in the highly permeable layer.

2.2 Specific Assumptions for the Axi-Symmetric Case

Assuming two concentric zones with different hydraulic transmissivity values (Figure 1), and a central circular hole of radius r_0, the liquid flow in the transmissive layer is radial and the problem is axi-symmetric. The radius of separation between the transmissive zones is called R_i, and the system considered is a cylinder of radius R_c. This cylinder contains, from top to bottom, all the layers presented in Figure 1. Assuming the soil is saturated, the boundary condition in the transmissive layer at $r = R_c$ is either (Touze-Foltz et al. 1999):

(a) zero flow

$$Q_r(R_c) = 0 \tag{3a}$$

$$h(R_c) \geq 0 \tag{3b}$$

(b) or, a specified head

$$h(R_c) = h_s \tag{4a}$$

$$Q_r(R_c) \geq 0 \tag{4b}$$

where: Q_r = radial rate of liquid flow in the transmissive layer; and h = hydraulic head in the transmissive layer. Field boundary conditions are obtained when both boundary conditions given by Equations 3 and 4 are satisfied for $h_s = 0$. This is the limit of validity of solutions that will be developed in Section 3.1.

The expression of the hydraulic head obtained in the case of a uniform hydraulic transmissivity is taken to be valid in the zones of the transmissive layer where the hydraulic transmissivity is uniform. As a consequence, based on the expression of the hydraulic head given by Touze-Foltz et al. (1999) for this latest case, the authors of the present paper obtained:

$$\begin{cases} h(r) = A_1 I_0(\alpha_1 r) + B_1 K_0(\alpha_1 r) - C & \text{for } (r_0 \leq r \leq R_i) \\ h(r) = A_2 I_0(\alpha_2 r) + B_2 K_0(\alpha_2 r) - C & \text{for } (R_i \leq r \leq R_c) \end{cases} \tag{5}$$

where: K_m and I_m = modified Bessel functions of the m^{th} order; A_n and B_n = constants of integration that must be determined from the boundary conditions; α_n is defined as (Rowe 1998):

$$\alpha_n = \sqrt{\frac{k_s}{(H_L + H_f)\theta_n}} \tag{6}$$

θ_n = hydraulic transmissivity of Zone n of the transmissive layer; and

$$C = H_L + H_f - h_a \tag{7}$$

Subscript 1 refers to Zone 1 of the transmissive layer surrounding the hole, whereas Subscript 2 refers to Zone 2 of the transmissive layer, remote from the hole. Thus, the boundary condition at $r = r_0$ is given by:

$$h_w + C = A_1 I_0(\alpha_1 r_0) + B_1 K_0(\alpha_1 r_0) \tag{8}$$

Continuity of the hydraulic head in the transmissive layer for $r = R_i$ may be expressed as:

$$A_1 I_0(\alpha_1 R_i) + B_1 K_0(\alpha_1 R_i) = A_2 I_0(\alpha_2 R_i) + B_2 K_0(\alpha_2 R_i) \tag{9}$$

The expression of the radial rate of liquid flow in the transmissive layer at the distance r from the circular hole axis, $Q_r(r)$, can be written in the form (Brown et al. 1987):

$$Q_r(r) = -2\pi r \theta \frac{\mathrm{d}h}{\mathrm{d}r} \tag{10}$$

where θ is the hydraulic transmissivity.

This leads to the following expression for continuity of the rate of liquid flow in the transmissive layer at $r = R_i$:

$$\alpha_1 \theta_1 \{A_1 I_1(\alpha_1 R_i) - B_1 K_1(\alpha_1 R_i)\} = \alpha_2 \theta_2 \{A_2 I_1(\alpha_2 R_i) - B_2 K_1(\alpha_2 R_i)\} \tag{11}$$

2.3 Specific Assumptions for the Case of a Hole in a Wrinkle

One can consider the case of a damaged rectilinear wrinkle of length L and width $2b$, with $L >> b$ so that the effects of leakage at the ends of the wrinkle can be neglected (Figure 2). It is assumed that the rate of liquid flow in the composite liner is not limited by the holes. The hole limiting case has been discussed by Rowe (1998) and Touze-Foltz et al. (1999). Liquid flow in the transmissive layer is assumed to be in the x-direction (Figure 2), normal to the longitudinal axis of the wrinkle. Under the assumption of two rectangular zones with different hydraulic transmissivity values parallel to the wrinkle in the transmissive layer, the problem of liquid flow becomes two-dimensional. The system considered is then a parallelepiped of width $2X_c$, *with a central wrinkle* as shown in Figure 2. This parallelepiped contains, from top to bottom, all the layers shown in Figure 2.

Boundary conditions considered at $x = X_c$ are (Touze-Foltz et al. 1999):

(a) zero flow

$$Q_x(X_c) = 0 \tag{12a}$$

$$h(X_c) \geq 0 \tag{12b}$$

(b) or, a specified head

$$h(X_c) = h_s \tag{13a}$$

$$Q_x(X_c) \geq 0 \tag{13b}$$

where: Q_x = rate of liquid flow in the transmissive layer in the direction normal to the longitudinal axis of the wrinkle; h = hydraulic head in the transmissive layer; and h_s = specified head at X_c. Field boundary conditions are obtained when both boundary conditions given by Equations 12 and 13 are satisfied for $h_s = 0$. This is the limit of validity of solutions that will be developed in Section 3.2.

It is assumed that the expression of the hydraulic head obtained by Touze-Foltz et al. (1999) for the case of a uniform hydraulic transmissivity is valid in the zones of the transmissive layer where the hydraulic transmissivity is uniform. As a consequence, based on the expression of hydraulic head obtained for a uniform hydraulic transmissivity, the hydraulic head distribution in the two zones is given by:

$$\begin{cases} h(x) = E_1 e^{-\alpha_1 x} + F_1 e^{\alpha_1 x} - C & \text{for } (b \leq x \leq X_i) \\ h(x) = E_2 e^{-\alpha_2 x} + F_2 e^{\alpha_2 x} - C & \text{for } (X_i \leq x \leq X_c) \end{cases} \tag{14}$$

where E_n and F_n are constants of integration that must be determined from the boundary conditions. Subscript 1 refers to Zone 1 of the transmissive layer adjacent to the wrinkle, and Subscript 2 refers to Zone 2 of the transmissive layer, remote from the wrinkle. The boundary condition for $x = b$ is given by:

$$(h_w + C) = E_1 e^{-\alpha_1 b} + F_1 e^{\alpha_1 b} \tag{15}$$

Continuity of the hydraulic head in the transmissive layer for $x = X_i$ gives:

$$E_1 e^{-\alpha_1 X_i} + F_1 e^{\alpha_1 X_i} = E_2 e^{-\alpha_2 X_i} + F_2 e^{\alpha_2 X_i} \tag{16}$$

The horizontal radial rate of liquid flow in the transmissive layer at the distance x from the middle of the wrinkle, $Q_x(x)$, can be expressed, by analogy with the axi-symmetric case, for one side of the wrinkle, as (Touze-Foltz et al. 1999):

$$Q_x(x) = -L\theta \frac{\mathrm{d}h}{\mathrm{d}x} \tag{17}$$

This leads to the following expression for the continuity of the horizontal rate of liquid flow in the transmissive layer at $x = X_i$:

$$\alpha_1 \theta_1 (-E_1 e^{-\alpha_1 X_i} + F_1 e^{\alpha_1 X_i}) = \alpha_2 \theta_2 (-E_2 e^{-\alpha_2 X_i} + F_2 e^{\alpha_2 X_i}) \tag{18}$$

3 HYDRAULIC HEAD PROFILE BELOW GEOMEMBRANE AND RATE OF LIQUID FLOW THROUGH COMPOSITE LINERS

3.1 General Solution for the Axi-Symmetric Case

3.1.1 *Solution for Zero Flow at $r = R_c$*

The boundary condition given by Equation 3a can now be expressed as:

$$A_{Q2} I_1(\alpha_2 R_c) - B_{Q2} K_1(\alpha_2 R_c) = 0 \quad (19)$$

Thus, in order to obtain the expression of the hydraulic head profile, one must solve Equations 8, 9, 11, and 19:

$$\begin{cases} A_{Q1} I_0(\alpha_1 r_0) + B_{Q1} K_0(\alpha_1 r_0) - (h_w + C) = 0 \\ A_{Q1} I_0(\alpha_1 R_i) + B_{Q1} K_0(\alpha_1 R_i) = A_{Q2} I_0(\alpha_2 R_i) + B_{Q2} K_0(\alpha_2 R_i) \\ \alpha_1 \theta_1 [A_{Q1} I_1(\alpha_1 R_i) - B_{Q1} K_1(\alpha_1 R_i)] = \alpha_2 \theta_2 [A_{Q2} I_1(\alpha_2 R_i) - B_{Q2} K_1(\alpha_2 R_i)] \\ A_{Q2} I_1(\alpha_2 R_c) - B_{Q2} K_1(\alpha_2 R_c) = 0 \end{cases} \quad (20)$$

where A_{Qn} and B_{Qn} are constants of integration. The subscript Q is related to the no-flow boundary condition. Substitution of B_{Q1} and B_{Q2} leads to:

$$\begin{cases} h(r) = A_{Q1}\left[I_0(\alpha_1 r) - \dfrac{I_0(\alpha_1 r_0) K_0(\alpha_1 r)}{K_0(\alpha_1 r_0)}\right] + (h_w + C)\dfrac{K_0(\alpha_1 r)}{K_0(\alpha_1 r_0)} - C \quad \text{for } (r_0 \le r \le R_i) \\ h(r) = A_{Q2}\left[I_0(\alpha_2 r) + \dfrac{I_1(\alpha_2 R_c) K_0(\alpha_2 r)}{K_1(\alpha_2 R_c)}\right] - C \quad \text{for } (R_i \le r \le R_c) \end{cases} \quad (21)$$

where:

$$A_{Q1} = \frac{-(h_w + C)[\alpha_2 \theta_2 \Pi^-_{1,1,0} + \alpha_1 \theta_1 \Pi^+_{0,1,1}](\alpha_2 R_i, \alpha_2 R_c, \alpha_1 R_i)}{\alpha_2 \theta_2 \Omega^-_{1,1}(\alpha_2 R_i, \alpha_2 R_c)\Omega^-_{0,0}(\alpha_1 R_i, \alpha_1 r_0) - \alpha_1 \theta_1 \Omega^+_{0,1}(\alpha_2 R_i, \alpha_2 R_c)\Omega^+_{1,0}(\alpha_1 R_i, \alpha_1 r_0)} \quad (22)$$

$$A_{Q2} = \frac{-\alpha_1 \theta_1 (h_w + C)\Pi^+_{0,1,1}(\alpha_1 R_i, \alpha_1 R_i, \alpha_2 R_c)}{\alpha_2 \theta_2 \Omega^-_{1,1}(\alpha_2 R_i, \alpha_2 R_c)\Omega^-_{0,0}(\alpha_1 R_i, \alpha_1 r_0) - \alpha_1 \theta_1 \Omega^+_{0,1}(\alpha_2 R_i, \alpha_2 R_c)\Omega^+_{1,0}(\alpha_1 R_i, \alpha_1 r_0)} \quad (23)$$

with:

$$\Omega^-_{n,m}(x, y) = I_n(x) K_m(y) - I_m(y) K_n(x) \quad (24)$$

$$\Omega^+_{n,m}(x, y) = I_n(x) K_m(y) + I_m(y) K_n(x) \quad (25)$$

$$\Pi^-_{n,m,l}(x, y, z) = \Omega^-_{n,m}(x, y) K_l(z) \quad (26)$$

$$\Pi^{+}_{n,\,m,\,l}(x, y, z) = \Omega^{+}_{n,\,m}(x, y)K_l(z) \tag{27}$$

Based on consideration of continuity of liquid flow, the total rate of liquid flow, Q, in the composite liner is equal to the sum of the rate of liquid flow into the soil liner below the hole ($r \le r_o$) and outside the hole ($r_o < r \le R_c$) and is given by:

$$Q = \pi r_0^2 k_s i_s - 2\pi r_0 \alpha_1 \theta_1 \left[\frac{A_{Q1}\Omega^{+}_{1,\,0}(\alpha_1 r_0, \alpha_1 r_0) - (h_w + C)K_1(\alpha_1 r_0)}{K_0(\alpha_1 r_0)}\right] \tag{28}$$

3.1.2 *Solution for Specified Head $h = h_s$ at $r = R_c$*

The boundary condition specifying the hydraulic head, h_s, at the end of the transmissive layer can be written as:

$$A_{p2} I_0(\alpha_2 R_c) + B_{p2} K_0(\alpha_2 R_c) - (h_s + C) = 0 \tag{29}$$

Thus, to get the hydraulic head distribution in the transmissive layer, the following system of Equations 8, 9, 11, and 29 must be solved:

$$\begin{cases} A_{p1} I_0(\alpha_1 r_0) + B_{p1} K_0(\alpha_1 r_0) - (h_w + C) = 0 \\ A_{p1} I_0(\alpha_1 R_i) + B_{p1} K_0(\alpha_1 R_i) = A_{p2} I_0(\alpha_2 R_i) + B_{p2} K_0(\alpha_2 R_i) \\ \alpha_1 \theta_1 [A_{p1} I_1(\alpha_1 R_i) - B_{p1} K_1(\alpha_1 R_i)] = \alpha_2 \theta_2 [A_{p2} I_1(\alpha_2 R_i) - B_{p2} K_1(\alpha_2 R_i)] \\ A_{p2} I_0(\alpha_2 R_c) + B_{p2} K_0(\alpha_2 R_c) - (h_s + C) = 0 \end{cases} \tag{30}$$

where A_{pn} and B_{pn} are constants of integration. The subscript p is related to the specified head boundary condition. Substitution of B_{p1} and B_{p2} gives:

$$\begin{cases} h(r) = A_{p1}\left[I_0(\alpha_1 r) - \dfrac{I_0(\alpha_1 r_0)K_0(\alpha_1 r)}{K_0(\alpha_1 r_0)}\right] + (h_w + C)\dfrac{K_0(\alpha_1 r)}{K_0(\alpha_1 r_0)} - C \quad \text{for } (r_0 \le r \le R_i) \\ h(r) = A_{p2}\left[I_0(\alpha_2 r) - \dfrac{I_0(\alpha_2 R_c)K_0(\alpha_2 r)}{K_0(\alpha_2 R_c)}\right] + (h_s + C)\dfrac{K_0(\alpha_2 r)}{K_0(\alpha_2 R_c)} - C \quad \text{for } (R_i \le r \le R_c) \end{cases} \tag{31}$$

where:

$$A_{p1} = \frac{\alpha_2\theta_2(h_s + C)\Pi^{+}_{0,\,1,\,0}(\alpha_2 R_i, \alpha_2 R_i, \alpha_1 r_0) - (h_w + C)[(\alpha_1\theta_1\Pi^{-}_{0,\,0,\,1} + \alpha_2\theta_2\Pi^{+}_{1,\,0,\,0})(\alpha_2 R_i, \alpha_2 R_c, \alpha_1 R_i)]}{\alpha_2\theta_2\Omega^{-}_{0,\,0}(\alpha_1 R_i, \alpha_1 r_0)\Omega^{+}_{1,\,0}(\alpha_2 R_i, \alpha_2 R_c) - \alpha_1\theta_1\Omega^{+}_{1,\,0}(\alpha_1 R_i, \alpha_1 r_0)\Omega^{-}_{0,\,0}(\alpha_2 R_i, \alpha_2 R_c)} \tag{32}$$

$$A_{p2} = \frac{(h_s + C)[(\alpha_1\theta_1\Pi^{+}_{1,\,0,\,0} + \alpha_2\theta_2\Pi^{-}_{0,\,0,\,1})(\alpha_1 R_i, \alpha_1 r_0, \alpha_2 R_i)] - \alpha_1\theta_1(h_w + C)\Pi^{+}_{0,\,1,\,0}(\alpha_1 R_i, \alpha_1 R_i, \alpha_2 R_c)}{\alpha_2\theta_2\Omega^{-}_{0,\,0}(\alpha_1 R_i, \alpha_1 r_0)\Omega^{+}_{1,\,0}(\alpha_2 R_i, \alpha_2 R_c) - \alpha_1\theta_1\Omega^{+}_{1,\,0}(\alpha_1 R_i, \alpha_1 r_0)\Omega^{-}_{0,\,0}(\alpha_2 R_i, \alpha_2 R_c)} \tag{33}$$

Based on consideration of continuity of liquid flow, the total rate of liquid flow in the composite liner is equal to the sum of the rate of liquid flow into the soil liner below the hole ($r \le r_o$) and outside the hole ($r_o < r \le R_c$) minus the rate of liquid flow escaping the transmissive layer for $r = R_c$, $Q_r(R_c)$:

$$Q_r(R_c) = -2\pi R_c \alpha_2 \theta_2 \left[\frac{A_{p2}\Omega^+_{1,0}(\alpha_2 R_c, \alpha_2 R_c) - (h_s + C)K_1(\alpha_2 R_c)}{K_0(\alpha_2 R_c)}\right] \tag{34}$$

This leads to the following expression for the rate of liquid flow through the hole in the geomembrane:

$$Q = \pi r_0^2 k_s i_s - 2\pi r_0 \alpha_1 \theta_1 \left[\frac{A_{p1}\Omega^+_{1,0}(\alpha_1 r_0, \alpha_1 r_0) - (h_w + C)K_1(\alpha_1 r_0)}{K_0(\alpha_1 r_0)}\right] \tag{35}$$

The rate of liquid flow, Q, is greater than the rate of liquid flow infiltrating into the soil liner, Q_s, because $Q_r(R_c)$ is greater than zero except for field boundary conditions as defined in Section 2.2. The expression for the rate of liquid flow, Q_s, is given by Equation 36:

$$Q = \pi r_0^2 k_s i_s - 2\pi \left[\begin{array}{l} \alpha_1 \theta_1 r_0 \dfrac{A_{p1}\Omega^+_{1,0}(\alpha_1 r_0, \alpha_1 r_0) - (h_w + C)K_1(\alpha_1 r_0)}{K_0(\alpha_1 r_0)} \\ -\alpha_2 \theta_2 R_c \dfrac{A_{p2}\Omega^+_{1,0}(\alpha_2 R_c, \alpha_2 R_c) - (h_s + C)K_1(\alpha_2 R_c)}{K_0(\alpha_2 R_c)} \end{array}\right] \tag{36}$$

3.2 Solution for the Two-Dimensional Case

3.2.1 *Solution for Zero Flow at $x = X_c$*

Following the expression of the hydraulic head given by Equation 13, the zero flow boundary condition given by Equation 12 can be expressed as:

$$-E_{Q2} e^{-\alpha_2 X_c} + F_{Q2} e^{\alpha_2 X_c} = 0 \tag{37}$$

Thus, to get the expression of the hydraulic head in the transmissive layer, the following system of Equations 15,16, 18, and 37 (expressing the continuity of hydraulic head and rate of liquid flow in the transmissive layer at $x = X_i$, and boundary conditions at $x = b$ and $x = X_c$) must be solved:

$$\begin{cases} E_{Q1} e^{-\alpha_1 b} + F_{Q1} e^{\alpha_1 b} - (h_w + C) = 0 \\ E_{Q1} e^{-\alpha_1 X_i} + F_{Q1} e^{\alpha_1 X_i} = E_{Q2} e^{-\alpha_2 X_i} + F_{Q2} e^{\alpha_2 X_i} \\ \alpha_1 \theta_1 (-E_{Q1} e^{-\alpha_1 X_i} + F_{Q1} e^{\alpha_1 X_i}) = \alpha_2 \theta_2 (-E_{Q2} e^{-\alpha_2 X_i} + F_{Q2} e^{\alpha_2 X_i}) \\ -E_{Q2} e^{-\alpha_2 X_c} + F_{Q2} e^{\alpha_2 X_c} = 0 \end{cases} \tag{38}$$

where E_{Qn} and F_{Qn} are constants of integration. The subscript Q refers to the no-flow boundary condition. Substitution of F_{Q1} and F_{Q2} leads to:

$$\begin{cases} h(x) = 2E_{Q1} e^{-\alpha_1 b} \sinh[\alpha_1 (b - x)] + (h_w + C) e^{\alpha_1 (x - b)} - C & \text{for } (b \le x \le X_i) \\ h(x) = 2E_{Q2} e^{-\alpha_2 X_c} \cosh[\alpha_2 (X_c - x)] - C & \text{for } (X_i \le x \le X_c) \end{cases} \tag{39}$$

where

$$E_{Q1} = \frac{e^{\alpha_1 X_i} (h_w + C) \{ \alpha_1 \theta_1 \cosh[\alpha_2 (X_c - X_i)] + \alpha_2 \theta_2 \sinh[\alpha_2 (X_c - X_i)] \}}{2 \{ \alpha_1 \theta_1 \cosh[\alpha_1 (b - X_i)] \cosh[\alpha_2 (X_c - X_i)] - \alpha_2 \theta_2 \sinh[\alpha_1 (b - X_i)] \sinh[\alpha_2 (X_c - X_i)] \}} \tag{40}$$

$$E_{Q2} = \frac{\alpha_1 \theta_1 (h_w + C) e^{\alpha_2 X_c}}{2 \{ \alpha_1 \theta_1 \cosh[\alpha_1 (b - X_i)] \cosh[\alpha_2 (X_c - X_i)] - \alpha_2 \theta_2 \sinh[\alpha_1 (b - X_i)] \sinh[\alpha_2 (X_c - X_i)] \}} \tag{41}$$

Based on consideration of continuity of liquid flow, the total rate of liquid flow Q in the composite liner is equal to the sum of the rate of liquid flow into the soil liner below the wrinkle ($x \le b$) and outside the wrinkle ($b < x \le X_c$) and is given by:

$$Q = 2L \left[b k_s i_s - \alpha_1 \theta_1 (h_w + C) \left\{ \frac{\tanh\left([\alpha_1 (b - X_i)] - \sqrt{\frac{\theta_2}{\theta_1}} \tanh[\alpha_2 (X_c - X_i)] \right)}{1 - \sqrt{\frac{\theta_2}{\theta_1}} \tanh([\alpha_1 (b - X_i)] \tanh[\alpha_2 (X_c - X_i)])} \right\} \right] \tag{42}$$

3.2.2 *Solution for Specified Head $h = h_s$ at $x = X_c$*

The boundary condition corresponding to a hydraulic head equal to h_s at $x = X_c$ gives:

$$E_{p2} e^{-\alpha_2 X_c} + F_{p2} e^{\alpha_2 X_c} - (h_s + C) = 0 \tag{43}$$

Thus, to obtain the expression of the hydraulic head, h, in the transmissive layer, the following system of Equations 15, 16, 18, and 43:

$$\begin{cases} E_{p1}e^{-\alpha_1 b} + F_{p1}e^{\alpha_1 b} - (h_w + C) = 0 \\ E_{p1}\,e^{-\alpha_1 X_i} + F_{p1}\,e^{\alpha_1 X_i} = E_{p2}\,e^{-\alpha_2 X_i} + F_{p2}\,e^{\alpha_2 X_i} \\ \alpha_1\theta_1(-E_{p1}e^{-\alpha_1 X_i} + F_{p1}e^{\alpha_1 X_i}) = \alpha_2\theta_2(-E_{p2}e^{-\alpha_2 X_i} + F_{p2}e^{\alpha_2 X_i}) \\ E_{p2}\,e^{-\alpha_2 X_c} + F_{p2}\,e^{\alpha_2 X_c} - (h_s + C) = 0 \end{cases} \tag{44}$$

is solved to give:

$$\begin{cases} h(x) = 2E_{p1}e^{-\alpha_1 b}\sinh[\alpha_1(b-x)] + (h_w + C)e^{\alpha_1(x-b)} - C \quad \text{for } (b \le x \le X_i) \\ h(x) = 2E_{p2}e^{-\alpha_2 X_c}\sinh[\alpha_2(X_c - x)] + (h_s + C)e^{\alpha_2(x - X_c)} - C \quad \text{for } (X_i \le x \le X_c) \end{cases} \tag{45}$$

where E_{Pn} and F_{Pn} are constants of integration. The subscript p refers to the specified head boundary conditions. Values of E_{P1} and E_{P2} are given by:

$$E_{p1} = \frac{e^{\alpha_1 b}}{2}\left[\frac{\alpha_2\theta_2(h_s + C) - (h_w + C)e^{\alpha_1(X_i - b)}\{\alpha_1\theta_1\sinh[\alpha_2(X_c - X_i)] + \alpha_2\theta_2\cosh[\alpha_2(X_c - X_i)]\}}{\alpha_2\theta_2\sinh[\alpha_1(b - X_i)]\cosh[\alpha_2(X_c - X_i)] - \alpha_1\theta_1\cosh[\alpha_1(b - X_i)]\sinh[\alpha_2(X_c - X_i)]}\right] \tag{46}$$

$$E_{p2} = \frac{e^{\alpha_2 X_c}}{2}\left[\frac{(h_s + C)e^{\alpha_2(X_i - X_c)}\{\alpha_1\theta_1\cosh[\alpha_1(b - X_i)] + \alpha_2\theta_2\sinh[\alpha_1(b - X_i)]\} - \alpha_1\theta_1(h_w + C)}{\alpha_2\theta_2\sinh[\alpha_1(b - X_i)]\cosh[\alpha_2(X_c - X_i)] - \alpha_1\theta_1\cosh[\alpha_1(b - X_i)]\sinh[\alpha_2(X_c - X_i)]}\right] \tag{47}$$

The total rate of liquid flow through the composite liner is given as the sum of the rate of liquid flow above the wrinkle plus the liquid flow of infiltration into the transmissive layer at a distance b from the wrinkle axis minus the rate of liquid flow escaping the transmissive layer at $x = X_c$, $Q_x(X_c)$:

$$Q_x(X_c) = -2L\alpha_2\theta_2[-2E_{p2}\,e^{-\alpha_2 X_c} + (h_s + C)] \tag{48}$$

This leads to the following expression for the rate of liquid flow through the hole in the geomembrane:

$$Q = 2L\left\{bk_s i_s - \alpha_1\theta_1[-2E_{p1}e^{-\alpha_1 b} + (h_w + C)]\right\} \tag{49}$$

The rate of liquid flow, Q, is greater than the rate of liquid flow infiltrating into the soil liner, Q_s , because $Q_x(X_c)$ is greater than zero except for field boundary conditions as defined in Section 2.3. The expression for the rate of liquid flow, Q_s, is given by Equation 50:

$$Q_s = 2L\left\{bk_s i_s - \alpha_1\theta_1\left[-2E_{p1}e^{-\alpha_1 b} + (h_w + C)\right] + \alpha_2\theta_2\left[-2E_{p2}e^{-\alpha_2 X_c} + (h_s + C)\right]\right\} \quad (50)$$

Analytical developments carried out in Sections 3.1 and 3.2 for the axi-symmetric and two-dimensional cases, respectively, could theoretically be extended to more complex hydraulic transmissivity fields, with n concentric zones with different hydraulic transmissivity values for the axi-symmetric case and n parallel zones for the two-dimensional case. However, expressions of the hydraulic head in the transmissive layer obtained in Equations 21, 31, 39, and 45 would be very complex, and numerical modelling would seem to be more appropriate for modelling more complex hydraulic transmissivity fields.

4 DEFINITION OF AN EQUIVALENT TRANSMISSIVITY

The tendency, when dealing with a nonuniform problem, is to find a method to return to a uniform problem. When dealing with composite liners exhibiting a variable hydraulic transmissivity, this can be done by defining an equivalent hydraulic transmissivity, θ_{eq}, as is usually done in the study of fractured media (Silliman 1987; Tsang 1992). The equivalent hydraulic transmissivity has been defined as the uniform hydraulic transmissivity, which, when applied under hydraulic boundary conditions identical to those applied to a given transmissivity field, reproduces the observed rate of liquid flow within a fractured medium. It follows from this definition that such a quantity can only be defined for a physical system for which the boundaries are well known and for boundary conditions allowing a flow at both ends of the system considered. Mathematical solutions given by Touze-Foltz et al. (1999) for the case of a uniform hydraulic transmissivity can be used to determine the equivalent hydraulic transmissivity for the simple geometries and transmissivity fields presented in the present paper, for axi-symmetric and two-dimensional problems. One has first to calculate the rate of liquid flow in a given composite liner using Equations 35 or 49, respectively, for the axi-symmetric and two-dimensional cases. Then the solutions for the case of a uniform hydraulic transmissivity developed by Touze-Foltz et al. (1999) are used to numerically extract the value of the equivalent hydraulic transmissivity, for the same geometry and boundary conditions.

It can be deduced from the work of Silliman (1987) that the equivalent hydraulic transmissivity only depends on the hydraulic transmissivity field and not on the hydraulic head for fractured media, where liquid flow only occurs in the fracture and not in the matrix surrounding it. In contrast, for composite liners, the liquid flow occurs both in the transmissive layer and in the soil liner. This leads to an equivalent transmissivity that depends on the hydraulic head at the two ends of the transmissive layer as will be shown in Section 5.5, where the relationship between equivalent hydraulic transmissivity and hydraulic head will be studied for the simple hydraulic transmissivity fields presented in the present paper, together with the influence of this relationship on the expected rates of liquid flow.

5 NUMERICAL INVESTIGATION OF THE PROPERTIES OF THE SOLUTIONS OBTAINED FOR A NONUNIFORM HYDRAULIC TRANSMISSIVITY FIELD

5.1 Evaluation of Head Loss at the Interface Between Two Zones With Different Hydraulic Transmissivity Values

As noted in Section 2, the continuous quantity at the interface between two zones having different hydraulic transmissivity values is the hydraulic pressure. Since the different hydraulic transmissivity values of the two zones are likely to be due to different thicknesses of the transmissive zones, the liquid velocity will be different on both sides of the interface and, hence, there will be a local head loss at the interface. This head loss must be calculated in two different ways depending on whether the first transmissivity is smaller or larger than the second. When the hydraulic transmissivity in Zone 1 of the transmissive layer, θ_1, is lower than the hydraulic transmissivity in Zone 2 of the transmissive layer, θ_2, the local head loss, Δh_{12}, can be calculated using the Carnot-Borda equation (Comolet 1961):

$$\Delta h_{12} = \frac{(V_1 - V_2)^2}{2g} \tag{51}$$

where: V_1 = liquid velocity in Zone 1 of the transmissive layer at $r = R_i$ or $x = X_i$; V_2 = liquid velocity in Zone 2 of the transmissive layer at $r = R_i$ or $x = X_i$; and g = gravitational acceleration.

When θ_1 is greater than θ_2, the local head loss, Δh_{12}, can be calculated using the following empirical formula (Pernès 2000):

$$\Delta h_{12} = \left[\frac{(s_1/s_2)^{1.8} - 1}{1 + 1.439(s_1/s_2)^{1.8}}\right]\frac{V_2^2}{2g} \tag{52}$$

where: s_1 = thickness of Zone 1 of the transmissive layer at $r = R_i$ or $x = X_i$; and s_2 = thickness of Zone 2 of the transmissive layer at $r = R_i$ or $x = X_i$.

To evaluate the potential importance of these local head losses, 5,000 different transmissivity fields were generated, for axi-symmetric and two-dimensional geometries. In both cases, the transmissive layer was divided into two zones of which the sizes were randomly generated such that the sum of their lengths was equal to $R_c - r_0$ or $X_c - b$, respectively, for the axi-symmetric and two-dimensional cases. For the axi-symmetric case, $r_0 = 10^{-3}$ m and $R_c = 1.3$ m; for the two-dimensional case, values of $b = 0.1$ m and $X_c = 2$ m were assumed.

One of the hydraulic transmissivity values, either θ_1 or θ_2, was set equal to 6.5×10^{-9} m^2s^{-1} and then the other value was selected from the following set {6.5×10^{-8} m^2s^{-1}; $6.5 \times$

10^{-7} m^{2}s^{-1}; 6.5×10^{-6} m^{2}s^{-1}; 6.5×10^{-5} m^{2}s^{-1}}, each being considered equiprobable. These hydraulic transmissivity values do not correspond to any definition of contact conditions. They have been chosen because they are simple multiples of 6.5×10^{-9} m^{2}s^{-1}. It is important to notice that according to the cubic law (Silliman 1987), an increase of one order of magnitude in the thickness of the transmissive layer (for example 2×10^{-5} to 2×10^{-4} m) results in an increase by a factor 1,000 in the transmissivity (6.5×10^{-9} to 6.5×10^{-6} m^{2}s^{-1}). Equation 51 was used to calculate the local head loss Δh_{12} when $\theta_1 < \theta_2$, whereas Equation 52 was used for $\theta_1 > \theta_2$. The maximum calculated local head losses, Δh_{12}, are presented in Tables 1 and 2 for the axi-symmetric and two-dimensional cases, respectively. Three different hydraulic heads on top of the composite liner, h_w, were examined: 0.3, 1, and 3 m. The boundary condition at $r = R_c$ or $x = X_c$ was taken to be zero hydraulic head ($h_s = 0$).

The results show that the local head losses calculated using Equations 51 and 52 are negligible as compared to the hydraulic heads applied on top of the composite liner. This validates the hypothesis, presented in Section 2, that there is quasi-continuity of the hydraulic head at the interface between two zones of different hydraulic transmissivity values.

Table 1. Maximal local head loss, Δh_{12}, for different leachate head values and hydraulic transmissivity conditions: axi-symmetric case.

Hydraulic transmissivity condition	Local head loss at the interface between Zones 1 and 2, Δh_{12} (m)		
	For h_w = 0.3 m (m)	For h_w = 1 m (m)	For h_w = 3 m (m)
$\theta_1 < \theta_2$	8.0×10^{-5}	8.5×10^{-4}	7.6×10^{-3}
$\theta_1 > \theta_2$	1.5×10^{-6}	1.6×10^{-5}	1.5×10^{-4}

Table 2. Maximal local head loss, Δh_{12}, for different leachate head values and hydraulic transmissivity conditions: two-dimensional case.

Hydraulic transmissivity condition	Local head loss at the interface between Zones 1 and 2, Δh_{12} (m)		
	For h_w = 0.3 m (m)	For h_w = 1 m (m)	For h_w = 3 m (m)
$\theta_1 < \theta_2$	6.9×10^{-6}	7.6×10^{-5}	6.8×10^{-4}
$\theta_1 > \theta_2$	2.1×10^{-4}	2.3×10^{-3}	2.1×10^{-2}

5.2 Influence of the Relative Size of Both Zones of the Transmissive Layer on the Rate of Liquid Flow

5.2.1 *Axi-Symmetric Case*

One of the new parameters introduced in the present paper as compared to previous analytical developments by Touze-Foltz et al. (1999) is the intermediate radius R_i or intermediate length X_i, for the axi-symmetric and two-dimensional cases, respectively. To evaluate the influence of R_i on the rate of liquid flow, calculations were conducted using the following parameters: $r_0 = 10^{-3}$ m, $H_L = 5$ m, $k_L = 10^{-9}$ ms^{-1}, and $h_w = 1$ m. The ratio of hydraulic transmissivity values was equal to 1,000. The adopted hydraulic transmissivity values were 1.6×10^{-8} and 1.6×10^{-5} m^{2}s^{-1}. Following from Rowe (1998), 1.6×10^{-8} m^{2}s^{-1} corresponds to good contact conditions as defined by Giroud and Bonaparte (1989) for a soil liner hydraulic conductivity equal to 10^{-9} ms^{-1}. The boundary condition at $r = R_c$ was $Q_r(R_c) = 0$ and $h_s = 0$, corresponding to field boundary conditions; R_c was calculated for each R_i. Figure 3 shows the influence of R_i/h_w on the ratio $Q/(h_w\theta_1)$. Two different curves were obtained: one for $\theta_1 < \theta_2$ and the other for $\theta_1 > \theta_2$.

For $\theta_1 < \theta_2$, the evolution of $Q/(h_w\theta_1)$ with R_i/h_w is limited. The value of $Q/(h_w\theta_1)$ decreases when R_i/h_w increases, for $R_i/h_w < 0.4$, and then becomes nearly independent of R_i/h_w. Thus, for $R_i/h_w > 0.4$, the size of Zone 1 has no significant influence on the rate of liquid flow, which becomes essentially proportional to θ_1. The decrease in

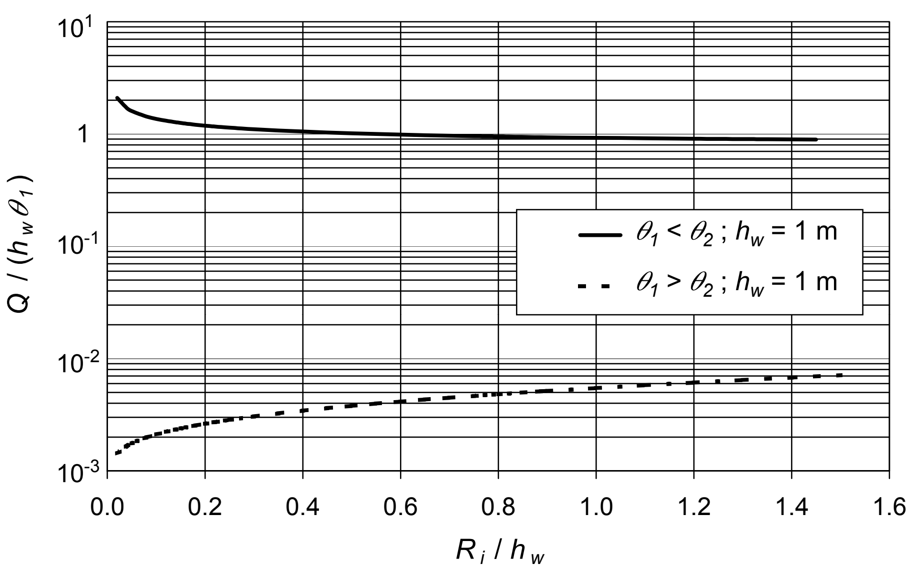

Figure 3. Variation of $Q/(h_w\theta_1)$ as a function of R_i/h_w for a hydraulic head $h_w = 1$ m for the axi-symmetric case.

$Q/(h_w\theta_1)$ with increasing R_i/h_w corresponds to the increase of the size of the transmissive layer zone with the lowest hydraulic transmissivity (around the hole), which results in a decrease in the rate of liquid flow.

For $\theta_1 > \theta_2$, the evolution of $Q/(h_w\theta_1)$ with R_i/h_w is more evident than in the previous case. The value of $Q/(h_w\theta_1)$ increases when R_i/h_w increases and no independence of $Q/(h_w\theta_1)$ with R_i/h_w is obtained even for values of R_i/h_w as high as 1.4. The increase in $Q/(h_w\theta_1)$ with increasing R_i/h_w corresponds to the increase of the size of the transmissive layer zone with the highest hydraulic transmissivity (around the hole), which leads to an increase in the rate of liquid flow. It can be concluded that, in the axisymmetric case, the influence of R_i on the rate of liquid flow is limited for θ_1 values less than θ_2 and is greater for θ_1 values greater than θ_2.

5.2.2 *Two-Dimensional Case*

The same phenomenon can be observed in Figure 4 for calculations carried out for the two-dimensional case, but the influence of X_i is still very large for values of X_i/h_w greater than 0.4, when θ_1 is less than θ_2. The following parameters were adopted for the calculations: $b = 10^{-1}$ m, $H_L = 5$ m, $k_L = 10^{-9}$ ms^{-1}, and $h_w = 1$ m; the hydraulic transmissivity ratio was equal to 1,000. The adopted hydraulic transmissivity values were 1.6×10^{-8} and 1.6×10^{-5} m^{2}s^{-1}. It can be seen in Figure 4 that the influence of X_i on the rate of liquid flow is limited for θ_1 greater than θ_2, but quite large for θ_1 less than θ_2. Consequently, the size of the transmissive layer that has to be described to

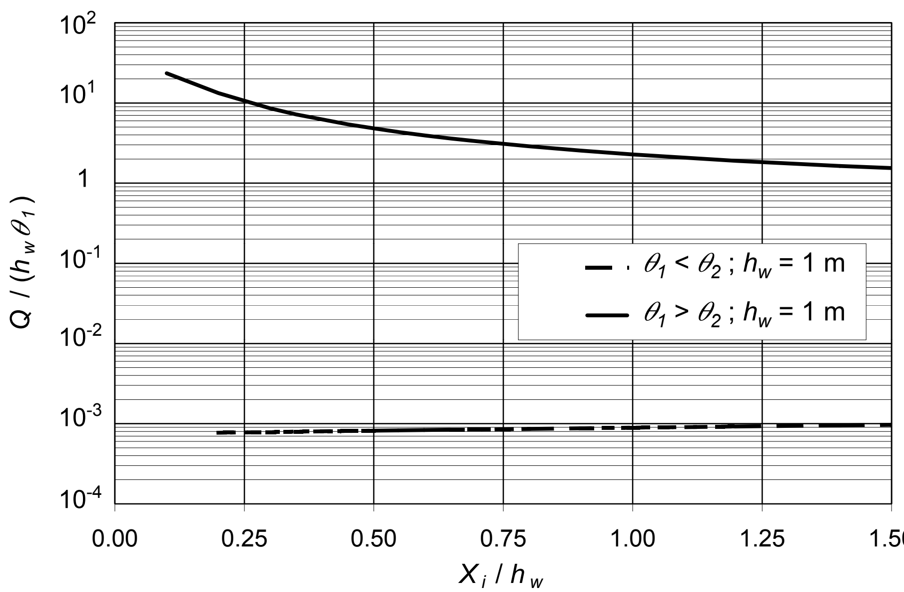

Figure 4. Variation of $Q/(h_w\theta_1)$ as a function of X_i/h_w for a hydraulic head $h_w = 1$ m for the two-dimensional case.

correctly evaluate the rate of liquid flow is higher than the one that has to be described for the axi-symmetric case, especially when θ_1 is less than θ_2. The main reason for this phenomenon is that head losses per unit length are greater in the axi-symmetric case than in the bi-dimensional case, due to radial dispersion of flow.

From these results, it appears that the acquisition of geometric data in the vicinity of the hole, or of the wrinkle, may not be sufficient to correctly estimate the rate of liquid flow through the composite liner.

5.3 Influence of the Transmissivity Field on the Rate of Liquid Flow

5.3.1 *Axi-Symmetric Case*

The second parameter introduced, when studying the influence of the variability of the hydraulic transmissivity, is the ratio of hydraulic transmissivity values in Zones 1 and 2 of the transmissive layer. Figure 5 shows the influence of this ratio on the non dimensional quantity $Q/(h_w\theta_1)$. Calculations were performed for $r_0 = 10^{-3}$ m, H_s = 5 m, $k_s = 10^{-9}$ ms^{-1}, h_w = 1 m, and R_i = 0.1 m. Values of θ_1 were either 6.5×10^{-9}, 1.6×10^{-8}, or 10^{-7} m^{2}s^{-1}. Following from Rowe (1998), the value of 10^{-7} m^{2}s^{-1} corresponds to poor contact conditions as defined by Giroud and Bonaparte (1989), and the value of 6.5×10^{-9} ms^{-1} was the hydraulic transmissivity given by Brown et al. (1987) for field boundary conditions, both for a soil liner hydraulic conductivity equal to 10^{-9} ms^{-1}. It can be seen that the influence of θ_1/θ_2 on $Q/(h_w\theta_1)$ is limited and, furthermore, for values of θ_1/θ_2 lower than 10^{-2} or greater than 10^2, the value of θ_2 no longer has any significant influence on $Q/(h_w\theta_1)$, and Q becomes proportional to h_w and θ_1.

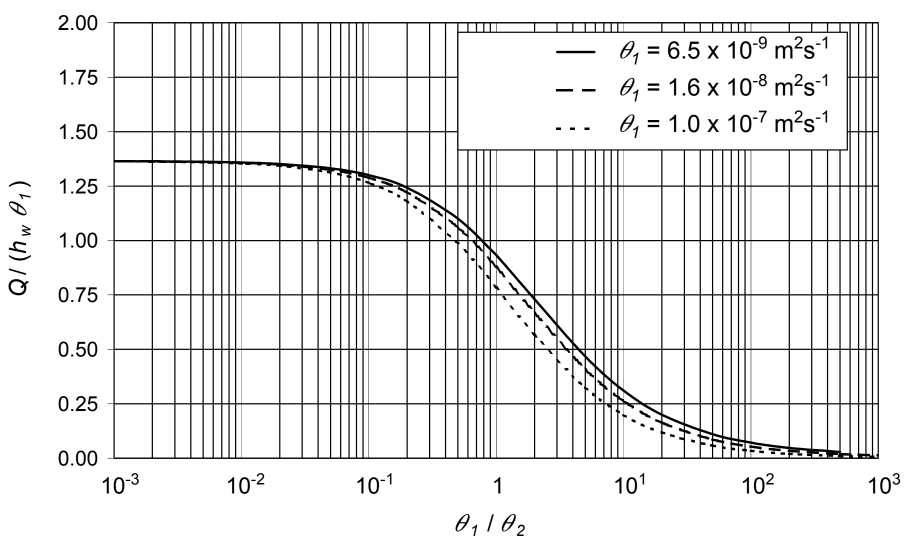

Figure 5. Variation of $Q/(h_w\theta_1)$ as a function of θ_1/θ_2 for the axi-symmetric case.

5.3.2 *Two-Dimensional Case*

Results obtained in the two-dimensional case show similar trends to those noted for the axi-symmetric case as shown on Figure 6. Calculations were performed for $b = 10^{-1}$ m, $H_s = 5$ m, $k_s = 10^{-9}$ ms^{-1}, $h_w = 1$ m, and $X_i = 0.4$ m. The main difference arises from the fact that the independence of $Q/(h_w\theta_l)$ with θ_l/θ_2 is only obtained for values of θ_l/θ_2 less than 10^{-4} or greater than 10^4.

5.4 Comparison of Rates of Liquid Flow Obtained With Uniform and Nonuniform Transmissivity Fields

To more completely investigate the influence of a hydraulic transmissivity field in the transmissive layer as compared to the uniform case, a statistical study, similar to the study described in Section 5.1, was conducted. The geometrical distributions and hydraulic transmissivity fields used to evaluate the local head loss at the interface between two zones with different hydraulic transmissivity values were used again. The main difference resulted from the fact that the boundary condition at the downstream side of the transmissive layer corresponded to field boundary conditions, i.e. $Q_r(R_c) = 0$ and $h_s = 0$ at $r = R_c$ for the axi-symmetric case, and $Q_x(X_c) = 0$ and $h_s = 0$ at $x = X_c$ for the two-dimensional case.

The values obtained in terms of rate of liquid flow were compared to the values obtained in the uniform case for the following five tested hydraulic transmissivity values: 6.5×10^{-9} m^{2}s^{-1}, 6.5×10^{-8} m^{2}s^{-1}, 6.5×10^{-7} m^{2}s^{-1}, 6.5×10^{-6} m^{2}s^{-1}, and 6.5×10^{-5} m^{2}s^{-1}. The results are presented in Tables 3 and 4 for the axi-symmetric and two-

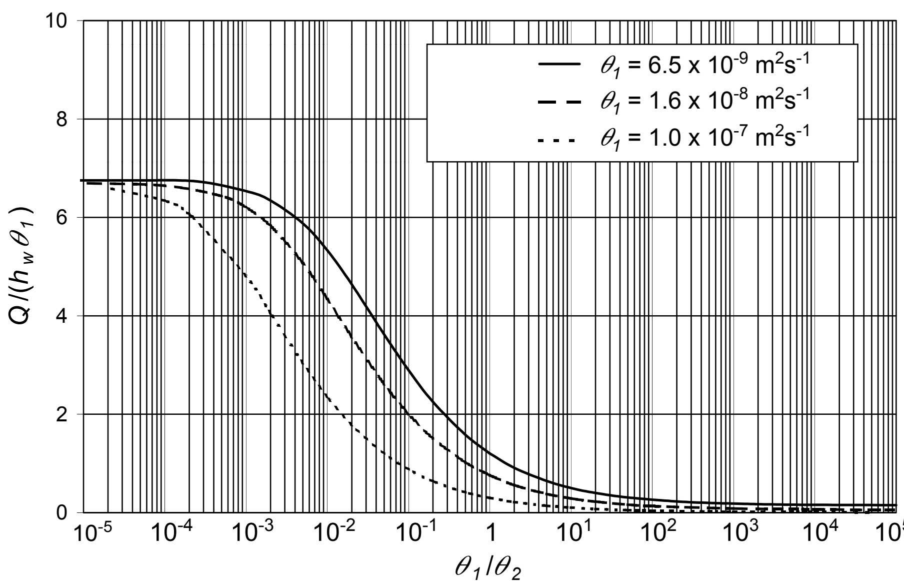

Figure 6. Variation of $Q/(h_w\theta_l)$ as a function of θ_l/θ_2 for the two-dimensional case.

dimensional cases, respectively. Here the rate of liquid flow, Q, is that calculated assuming a uniform hydraulic transmissivity equal to θ_I, and the results are presented in terms of probability that the given fraction of this flow, Q, underestimates the rate of liquid flow obtained taking into account two zones with different hydraulic transmissivity values. Thus, for example, Table 3 indicates that the rate of liquid flow, Q, calculated from the uniform solution for $\theta_I = 6.5 \times 10^{-9}$ m^{2}s^{-1} always underestimates the flow calculated considering the nonuniform transmissivity and that the maximum rate of liquid flow obtained is always less than 10Q. Table 3 also indicates that the probability of Q exceeding the calculated rate of liquid flow (considering the nonuniform transmissivity) is approximately 40% ((1.0 – 6.04 × 10^{-1}) × 100) for $\theta_I = 6.5 \times 10^{-8}$ m^{2}s^{-1}, and 62% ((1.0 – 3.82 × 10^{-1}) × 100) for $\theta_I = 6.5 \times 10^{-6}$ m^{2}s^{-1}. In the latter case, the rate of liquid flow is always less than 5Q.

Table 3. Probability that a given fraction of the rate of liquid flow obtained with a uniform hydraulic transmissivity equal to θ_I underestimates the rate of liquid flow obtained taking into account two zones with different hydraulic transmissivity values: axisymmetric case.

Flow rate	θ_I				
	6.50×10^{-9}	6.50×10^{-8}	6.50×10^{-7}	6.50×10^{-6}	6.50×10^{-5}
Q/5,000	1	1	1	1	1
Q/2,000	1	1	1	1	9.94×10^{-1}
Q/1,000	1	1	1	1	9.54×10^{-1}
Q/500	1	1	1	1	8.35×10^{-1}
Q/300	1	1	1	9.97×10^{-1}	8.02×10^{-1}
Q/200	1	1	1	9.84×10^{-1}	7.73×10^{-1}
Q/100	1	1	1	9.27×10^{-1}	5.89×10^{-1}
Q/50	1	1	1	8.09×10^{-1}	5.86×10^{-1}
Q/30	1	1	9.87×10^{-1}	7.84×10^{-1}	5.51×10^{-1}
Q/20	1	1	9.73×10^{-1}	7.41×10^{-1}	4.07×10^{-1}
Q/10	1	1	9.10×10^{-1}	5.88×10^{-1}	3.83×10^{-1}
Q/5	1	9.99×10^{-1}	7.82×10^{-1}	5.86×10^{-1}	3.75×10^{-1}
Q/2	1	9.56×10^{-1}	5.91×10^{-1}	3.82×10^{-1}	1.94×10^{-1}
Q	1	6.04×10^{-1}	5.75×10^{-1}	3.82×10^{-1}	0
2Q	1.07×10^{-2}	8.21×10^{-3}	9.48×10^{-3}	2.04×10^{-3}	0
5Q	9.76×10^{-4}	2.05×10^{-3}	0	0	0
10Q	0	0	0	0	0

Table 4. Probability that a given fraction of the rate of liquid flow obtained with a uniform hydraulic transmissivity equal to θ_l underestimates the rate of liquid flow obtained taking into account two zones with different hydraulic transmissivity values: two-dimensional case.

Flow rate	θ_l				
	6.5×10^{-9}	6.5×10^{-8}	6.5×10^{-7}	6.5×10^{-6}	6.5×10^{-5}
Q/100	1	1	1	1	1
Q/50	1	1	1	1	8.24×10^{-1}
Q/30	1	1	1	1	7.87×10^{-1}
Q/20	1	1	1	9.14×10^{-1}	5.89×10^{-1}
Q/10	1	1	1	7.84×10^{-1}	5.89×10^{-1}
Q/5	1	1	8.05×10^{-1}	5.92×10^{-1}	4.11×10^{-1}
Q/2	1	8.60×10^{-1}	6.01×10^{-1}	3.94×10^{-1}	2.25×10^{-1}
Q	1	7.83×10^{-1}	3.92×10^{-1}	3.94×10^{-1}	0
2Q	1.07×10^{-1}	4.39×10^{-1}	3.92×10^{-1}	1.98×10^{-1}	0
5Q	2.87×10^{-2}	8.03×10^{-2}	1.68×10^{-1}	0	0
10Q	1.44×10^{-2}	1.81×10^{-2}	0	0	0
20Q	5.13×10^{-3}	5.02×10^{-3}	0	0	0
30Q	1.03×10^{-3}	0	0	0	0
100Q	0	0	0	0	0

From Tables 3 and 4, it can be deduced that, for $\theta_l = 6.5 \times 10^{-9}$ m^{2}s^{-1}, using the solution obtained in the uniform case will underestimate the rate of flow that would be obtained using a solution that takes into account two different hydraulic transmissivity values in the transmissive layer. For the other values of θ_l examined, it will usually overestimate the rate of flow by as much as two orders of magnitude for the highest θ_l values.

5.5 Relevance of the Definition of an Equivalent Hydraulic Transmissivity

To study the relevance of the definition of an equivalent hydraulic transmissivity, the geometrical distributions and hydraulic transmissivity fields described for the studies presented in Sections 5.1 and 5.4 have been used again in this section. Values of R_c and X_c are as given in Section 5.1, thus, the liner geometry is fixed, except the geometry of the transmissive layer. Equivalent hydraulic transmissivity values have been calculated for three different hydraulic heads: 0.3, 1, and 3 m, for the 5,000 geometric distributions examined, following the protocol described in Section 4. The equivalent hydraulic transmissivity values obtained for these three hydraulic heads, $\theta_{eq}(h_w = 0.3$ m), $\theta_{eq}(h_w$ 1 m), and $\theta_{eq}(h_w = 3$ m) were different for a given composite liner. To evaluate the influence of this difference on the rate of liquid flow, the following procedure was followed. First, the rate of liquid flow was calculated for each of these three equivalent transmissivity values with a hydraulic head equal to 0.3 m. Then, the relative differences in the rate of liquid flow defined as:

$$DQ_1 = \frac{Q[\theta_{eq}(h_w = 0.3 \text{ m})] - Q[\theta_{eq}(h_w = 1 \text{ m})]}{Q[\theta_{eq}(h_w = 0.3 \text{ m})]} \tag{53}$$

$$DQ_2 = \frac{Q[\theta_{eq}(h_w = 0.3 \text{ m})] - Q[\theta_{eq}(h_w = 3 \text{ m})]}{Q[\theta_{eq}(h_w = 0.3 \text{ m})]} \tag{54}$$

were calculated for all of the 5,000 configurations tested. Values obtained for DQ_1 varied between 0 and 13% and values of DQ_2 between 0 and 17%. Considering R_1 values, approximately 50% of the values obtained were less than 1%, as shown in Figure 7, but there is a non-negligible number of configurations for which DQ_1 can be as high as 13%. The same conclusion can be drawn for DQ_2.

Consequently, there is a non-negligible number of configurations for which the equivalent hydraulic transmissivity is so greatly dependent on the hydraulic head that it results in a non-negligible effect on the calculated rate of liquid flow. Thus, it is recommended that the notion of equivalent hydraulic transmissivity be viewed with great caution when dealing with composite liners. This conclusion is considered particularly apt given that the equivalent hydraulic transmissivity can not be defined for field boundary conditions where the value of R_c or X_c and the hydraulic transmissivity field are strongly dependent on each other.

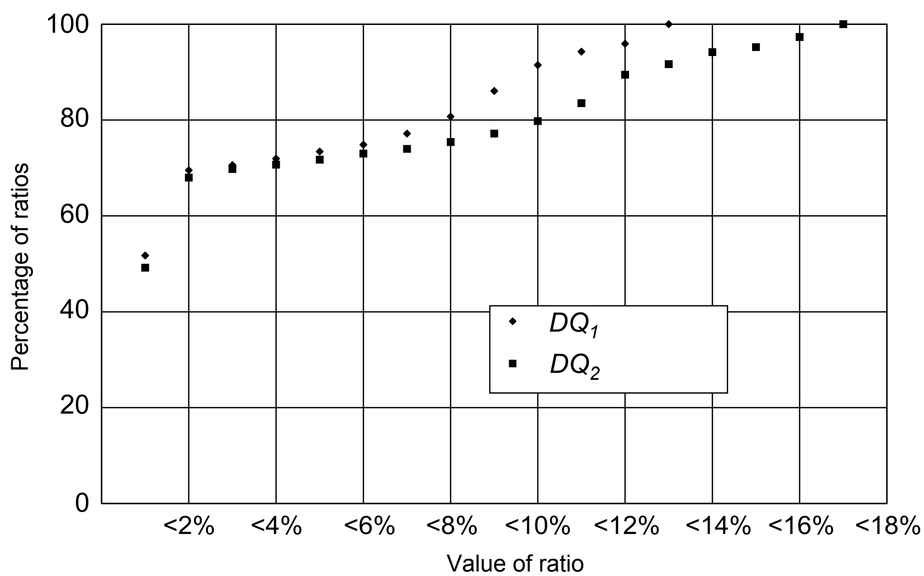

Figure 7. Distribution of ratios R_1 and R_2 for the 5,000 tested configurations.

6 CONCLUSION

A general framework for calculating the rate of liquid flow through composite liners for the case where there is a nonuniform hydraulic transmissivity has been presented. The results of the numerical study conducted in Section 5 demonstrate that the new parameters introduced for the case of variable hydraulic transmissivity have a non-negligible effect on the rate of liquid flow. For instance, the study of the influence of the relative size of the two zones of the transmissive layer on the rate of liquid flow has demonstrated the need to carefully describe the geometry in a large area around the hole in the geomembrane. In addition, calculations carried out with different hydraulic transmissivity ratios make it clear that the thickness of the transmissive layer is a key parameter. Furthermore, calculations have shown that, while solutions obtained for uniform hydraulic transmissivity cases will rarely underestimate the rate of liquid flow for nonuniform hydraulic transmissivity cases (assuming the transmissivity near the hole is adequately defined), these solutions can overestimate the rate by as much as two orders of magnitude for the range of cases considered. These results suggest the need for continuing research into the description of the transmissive layer geometry and in its relation to rates of liquid flow through composite liners.

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NOTATIONS

Basic SI units are given in parentheses.

A_1, A_2 = constants (dimensionless)

A_{p1}, A_{p2} = coefficients for specified head boundary condition, values depending on boundary conditions (dimensionless)

A_{Q1}, A_{Q2} = coefficients for zero flow boundary condition, values depending on boundary conditions (dimensionless)

B_1, B_2 = constants (dimensionless)

B_{p1}, B_{p2} = coefficients for specified head boundary condition, values depending on boundary conditions (dimensionless)

B_{Q1}, B_{Q2} = coefficients for zero flow boundary condition, values depending on boundary conditions (dimensionless)

b = half width of a wrinkle (m)

C = $H_L + H_f - h_a$ (m)

DQ_1 = ratio of rates of liquid flow (dimensionless)

DQ_2	=	ratio of rates of liquid flow (dimensionless)
E_1 , E_2	=	constants (m)
E_{p1} , E_{p2}	=	coefficients for specified head boundary condition, values depending on boundary conditions (m)
E_{Q1} , E_{Q2}	=	coefficients for zero flow boundary condition, values depending on boundary conditions (m)
F_1 , F_2	=	constants (m)
F_{p1} , F_{p2}	=	coefficients for specified head boundary condition, values depending on boundary conditions (m)
F_{Q1} , F_{Q2}	=	coefficients for zero flow boundary condition, values depending on boundary conditions (m)
g	=	gravitational acceleration (ms^{-2})
H_f	=	thickness of foundation layer (m)
H_L	=	thickness of soil liner (CCL or GCL) (m)
h	=	hydraulic head in the transmissive layer (m)
h_a	=	potentiometers head in an aquifer or at bottom of foundation layer (m)
h_s	=	specified hydraulic head in transmissive layer at $r = R_c$ (m)
h_w	=	leachate head acting on top of geomembrane (m)
I_m	=	modified Bessel function of m^{th} order (dimensionless)
i_s	=	maximum mean gradient across soil liner and foundation layer (dimensionless)
K_m	=	modified Bessel function of m^{th} order (dimensionless)
k_f	=	hydraulic conductivity of foundation layer (ms^{-1})
k_L	=	hydraulic conductivity of soil liner (GCL or CCL) (ms^{-1})
k_s	=	harmonic mean hydraulic conductivity of soil liner and foundation layer (ms^{-1})
L	=	length of wrinkle (m)
Q	=	rate of liquid flow through hole in geomembrane (m^3s^{-1})
Q_r	=	radial rate of liquid flow in transmissive layer for circular problem (m^3s^{-1})
Q_s	=	rate of liquid flow into soil (soil liner + foundation layer) (m^3s^{-1})
Q_x	=	rate of liquid flow in transmissive layer for two-dimensional problem (m^3s^{-1})
R_c	=	physical radius of system studied in axi-symmetric case (m)

R_i	=	radius of separation of Zones 1 and 2 of transmissive layer (m)
r	=	radial distance (m)
r_0	=	radius of hole in geomembrane (m)
s_1	=	thickness of Zone 1 of transmissive layer for $r = R_i$ or $x = X_i$ (m)
s_2	=	thickness of Zone 2 of transmissive layer for $r = R_i$ or $x = X_i$ (m)
V_1	=	mean fluid velocity in Zone 1 of transmissive layer for $r = R_i$ or $x = X_i$ (m)
V_2	=	mean fluid velocity in Zone 2 of transmissive layer for $r = R_i$ or $x = X_i$ (m)
X_c	=	width of cell or system studied in case of damaged wrinkle (m)
X_i	=	width of separation of Zones 1 and 2 of transmissive layer (m)
x	=	horizontal distance (m)
α_n	=	parameter defined by Equation 6 (m^{-1})
Δh_{12}	=	local head loss at interface between Zones 1 and 2 of transmissive layer (m)
$\Pi^-_{n, m, l}$	=	parameter defined by Equation 26 (dimensionless)
$\Pi^+_{n, m, l}$	=	parameter defined by Equation 27 (dimensionless)
θ	=	hydraulic transmissivity (m^2s^{-1})
θ_{eq}	=	equivalent hydraulic transmissivity (m^2s^{-1})
θ_n	=	hydraulic transmissivity of n^{th} zone of transmissive layer (m^2s^{-1})
$\Omega^-_{n, m}$	=	parameter defined by Equation 24 (dimensionless)
$\Omega^+_{n, m}$	=	parameter defined by Equation 25 (dimensionless)