

## **EFFECT OF BACKFILL EROSION ON MOMENTS IN BURIED RIGID PIPES**

Zheng TAN, Masters Student, zheng@ce.queensu.ca

and

Ian D. MOORE, Professor and Canada Research Chair in Infrastructure Engineering

moore@civil.queensu.ca 613 533 3160

### **ABSTRACT**

The service life of rigid sewer pipes is often controlled by joint integrity. Leakage at the joint can result in ingress of water and the development of a void where soil has been eroded from beside the pipe. The influence of soil voids on the stability of buried rigid pipes is investigated, considering the effects of void size, void location and void shape. A series of simplified void geometries are defined, and elastic and elastic-plastic finite element analyses are performed to study how those voids influence bending moments in the rigid sewer. Erosion voids at the springlines lead to increased bending moments at crown, invert and springlines. Elastic analysis indicates that the bending moments approximately double once the void contacts the external surface of the pipe over a 90 degree arc. If shear failure is included and the loosened backfill is modelled, the moments approximately triple for that void geometry. In contrast, a void under the invert leads to decreases in the magnitude of bending moments, and for large void size, the moments can reverse sign. The moment increases slowly when a void starts to grow at the springline, but these moment changes accelerate once the void contacts the pipe over a 45 degree arc. This preliminary study suggests that efforts to arrest the growth of erosion voids at

the springlines should be undertaken before the voids reach this size. All results presented are theoretical in nature, and physical testing is needed to evaluate the performance of these calculations.

Words: 2750, 1 Table, and 9 figures

## 1 INTRODUCTION

Soil-pipe interaction research has resolved a variety of buried pipe limit states. Major advances in the 1960's and 1970's resulted from the development of soil-pipe interaction solutions that permitted rational calculation of thrust, moment and deformation (e.g. the closed form solution of Hoeg [1], and the finite element procedure of Katona [2]). Stability limit states such as buckling (Moore [3]), local bending in the profile and around the circumference (Moore and Hu [4]), Dhar et al. [5]), and local buckling (Dhar and Moore [6]) have all been investigated. Based on these advances, it might seem that the final challenges associated with analysis and engineering of the buried pipe problem have been resolved.

Now, durability relates to the ability of a pipe to withstand, to a satisfactory degree, the effects of service conditions (see NRCC [7]). This has generally been interpreted as the ability of the pipe to resist wear and decay, and investigations to ensure service life have generally worked to ensure that sufficient pipe material of sufficient durability has been employed. However, the growing need to repair and replace aging culvert and sewer infrastructure has made it clear that durability is not simply a function of the long term performance of the structural material. Rather, many rigid pipes (i.e. vitrified clay and reinforced concrete) fail as a result of joint leakage, ingress of groundwater, and the accompanying erosion of the soil envelope adjacent to the pipe. Workers concerned with pipe repair using liners have raised important questions regarding the impact of those soil voids on the repaired pipe (e.g. [8]).

A program of research at Queen's University has recently been initiated to study the physical processes and implications of deterioration of the backfill soil. Computer studies are being undertaken to design a laboratory test program, and to provide preliminary guidance on the acceptable limits of soil erosion. The study reported here quantifies the impact of soil voids on the bending moments in buried rigid pipes. Idealised soil voids are defined, and two dimensional (plane strain) finite element analysis is used to assess the impact of those voids on bending moments. Voids at the springlines and the invert are studied, to determine moment amplification with increases in the extent of the void. While this study could be used to provide preliminary guidance on the stability of existing structures with such erosion voids, it will be followed by experimental studies to assess the reliability of the computer analysis and to determine how soil erosion and void growth control pipe failure.

## 2 THE RIGID PIPE

A rigid pipe may be defined as one which, under its maximum load, does not deform sufficiently to produce significant changes in the earth pressures applied by the soil in which it is laid. Rigid pipes support loads in the ground by virtue of the resistance of the pipe as a ring to bending. For example, design of steel reinforcement in concrete pipe is undertaken to ensure the pipe can support the expected bending moments.

Now, while pipe deformations in rigid pipes are not sufficient to influence the bending moments that develop, the role of the envelope of soil surrounding the pipe is nevertheless critical. The concrete pipe does not simply act as a stiff ring under the action

of vertical forces at crown and invert (the so-called 'parallel plate' loading condition). Rather, earth pressures are distributed across the top and bottom halves of the pipe, and lateral earth pressures also develop on the sides of the structure. It is actually the difference between the vertical and horizontal earth pressures that produces bending moments, and 'Bedding Factor'  $BF$  is used in reinforced concrete pipe design to quantify the reductions in bending moment relative to the case of stiff ring under parallel plate loading. Use of values of  $BF$  exceeding 4 for design of pipes surrounded by good quality backfill is an indication of the substantial reductions in bending moments that occur if the pipe is placed within an envelope of good quality backfill.

A rigid pipe of wall thickness  $t = 44\text{mm}$  and outer diameter  $OD = 388\text{mm}$  is studied, to illustrate the effects of void creation on pipe stability. The pipe is specified with Young's Modulus of  $E = 28\text{GPa}$ , a reasonable value for reinforced concrete. However, increases or decreases in this value have little effect on the bending moments that are calculated, as long as this modulus remains orders of magnitude higher than the surrounding soil. The strength of this rigid pipe will be assumed to be defined by a capacity to resist vertical forces under two or three point loading of  $23\text{kN/m}$ . A zero value of Poisson's ratio is used, assuming that the presence of joints releases constraint in the axial direction. Again, however, the use of other values will have little if any effect on the calculated bending moments.

### 3 NUMERICAL MODELING – CONCRETE PIPE ALONE

Rigid pipe structural properties in the circumferential direction include the effective elastic modulus of the pipe wall modulus ( $E_p$ ), the cross-section area per unit length of pipelines ( $A_p$ ) and the second moment of area per unit length ( $I$ ). For a long plain pipe of wall thickness  $t$

$$A_p = t \quad (1)$$

$$I = \frac{t^3}{12} \quad (2)$$

For this rigid pipe with its load capacity under two point (i.e. parallel plate) loading of  $F = 23$  kN/m, the moment at the crown,  $M_{cr}$ , and the extreme fiber tensile stress at that location  $\sigma_{cr}$  (stress) can be calculated as follows, Moore [9]:

$$M_{cr} = Fr\pi^{-1} \quad (3)$$

$$\sigma = \frac{Mt}{2I} \quad (4)$$

Results of these ‘closed form’ calculations are included on Table 1.

The response of the rigid pipe has also been examined using the finite element program AFENA, Carter et al. [10]. This calculation was performed to assess whether the numerical analysis correctly provides the stress and moment in the reinforced concrete pipe. Pipe properties identical to those reported in Table 1 were used.

The mesh was defined for half of the RC pipe, using the symmetry of the problem, Figure 1a. Two hundred and forty triangular elements, each with six nodes, were used to model the right hand half of the RC pipe. The calculated distributions of  $\sigma_{xx}$  and  $\sigma_{yy}$  are shown on Figures 1b and 1c (like all stress results presented in this article, shown using a compression positive sign convention). The results of compressive stress and bending moment at the crown of the RCP are 3.95 MPa and 23 kN.m/m, respectively. These are almost identical to the results from the closed form calculation, demonstrating that the finite element procedure is giving correct solutions. The rest of this article focuses on this pipe when buried, quantifying the impact of the voids on the bending moments and tensile stresses at the extreme fibers.

#### 4 ELASTIC CALCULATIONS FOR VOIDS AT SPRINGLINES

Figure 2 shows the geometries for a series of voids that were defined at the springlines. These are expected where water flows into the pipe from groundwater through fractures at the culvert springlines, and from the haunches or shoulders through leaking joints. While there are a wide range of potential void geometries (and they are not expected to have a regular shape), circular shapes were chosen for this first study of this phenomenon so that the geometry is clear. The void geometries were defined so that:

- a. Voids are approximated as having uniform dimension along the pipe. This is associated with the use of two dimensional plane strain analysis of soil-void-pipe system (erosion associated with flow through pipe joints means that real voids would likely have three dimensional geometry, so the analysis here corresponds to some ‘average’ void geometry along one specific pipe segment).

- b. Each 'set' of three voids has three different diameters; the smallest contacts the springline over an arc of approximately  $30^\circ$  (actually  $29.58^\circ$ ), the next intersects over  $60^\circ$  (actually  $59.66^\circ$ ), and the largest void intersects the pipe over a  $90^\circ$  arc (actually  $90.84^\circ$ ). The angles provided in parentheses are somewhat different to the target values, since the angle choices are constrained by the finite number of nodes used around the external pipe circumference.
- c. Three different void sets are defined, as shown in Figures 2a to 2c; these allow the effect of the lateral extent of the voids to be studied (from wide, Figure 2a, to narrow, Figure 2c)
- d. Traditionally, theoretical pipe solutions have been developed for two idealized interface conditions: perfect adhesion of the soil to the structure (the bonded or no-slip interface condition), and zero adhesion (the full-slip or smooth interface condition), e.g. [1,9]. Since the actual pipe response in the laboratory or in the field is expected to lie somewhere between these two limits, both are studied and reported here.

Each analysis was performed simplifying the initial burial conditions by considering the pipe within a uniform backfill (neglecting nonuniformities associated with burial details, such as SIDD earth pressure distributions). The analysis represents the initial loading conditions as a surface pressure from overlying soil of 100kPa (equivalent to 5m of typical compacted backfill). These approximations are used to isolate the effect of the non-uniform loadings associated with void formation, from the nonuniformities that can occur during conventional pipe burial.

Finite element analysis was used to calculate the stress distribution across the wall of the rigid pipe, Figures 3 to 5. These figures show the results of elastic soil-structure interaction analysis at the crown, the invert and the springlines respectively (elastic-plastic analysis is considered in a subsequent section). The initial stresses (where there is no void formation) feature bending stresses at the extreme fibers that substantially exceed the average compressive stresses associated with hoop force. Each distribution of stress across the pipe wall is almost linear. Deviations from the linear result from the fact that these structures are ‘thick rings’, and do not exactly satisfy the kinematics and constitutive approximations associated with conventional thin ring theory.

These linear elastic calculations, performed for the widest void geometries (Figure 2a), indicate how increases in the diameter of the void (or the angle  $\alpha$  over which the void contacts the exterior of the pipe), increase the magnitude of the extreme fiber stresses and the bending moments at all key locations (the crown, the invert and the springlines). Pipe burial depth (or the overburden pressure discussed previously) has been chosen so that factor of safety against tensile cracking is initially about 2. If this rigid pipe has tensile cracking stress of 4MPa (the value implied by the analysis presented in Table 1), it could be expected to reach that limit state first at the springline, closely followed at the crown and invert. This would occur when the void approaches the largest size shown in Figure 2a (a void contacting the pipe over an arc somewhat less than  $90^\circ$ ).

Figure 6 summarises the percentage increase in extreme fiber tension with growth in the void-pipe contact angle. These results indicate that the wider cavities (i.e. void set A)

produce higher tensile stresses relative to the very narrow voids (i.e. void set C). However, the dominant geometrical factor appears to be the angle of void in contact with the RCP.

A comparison between solutions for the two different interface conditions indicates that the growth of a void generally has more effect on stresses when the interface is rough (or bonded).

## 5 ELASTIC-PLASTIC CALCULATIONS FOR VOIDS AT SPRINGLINES

The introduction of a cavity in the soil adjacent to the pipe could reasonably be expected to induce shear failure in the soil, particularly for cases where the backfill is granular and it has little if any cohesion. The model used for the soil was therefore extended to include the influence of shear failure. Backfill soil strength was defined using a friction angle  $\phi$  of  $20^\circ$  (and small cohesion of 0.002kPa to remove spurious failure of points at zero stress). Modulus is set to 2MPa and Poisson's ratio is set to 0.333. These strength and modulus choices represent what might be expected for very loose soil, to reflect significant dilation and weakening of the backfill in the vicinity of the void (given that water is present and erosion is occurring). Figure 7 shows the stress comparison at crown and springline between elastic analysis and elastic-plastic analysis for bonded interface, and Figure 8 for smooth interface.

Figure 9 provides a summary of the resulting changes in tensile and compressive stress at the extreme fibers at the crown of the pipe. A comparison of these results with those

provided in Figure 6 makes it clear that the incorporation of shear failure into the analysis substantially increases the magnitude of stress enhancement associated with void formation. For the largest voids (with an angular contact over a  $90^\circ$  arc across the springline), the stresses are enhanced over 200%. Therefore, while shear failure does not significantly change bending stresses or the likelihood of fracture for pipes buried in continuous (void free) backfill, it appears that shear failure in the soil substantially enhances the impact of voids on the demand placed on the buried pipe. This tripling of stress at the extreme fiber should be more than enough to induce tensile fracture in a typical reinforced concrete pipe buried near its depth limit.

## 6 ELASTIC-PLASTIC CALCULATIONS FOR VOIDS AT INVERT

The impact of void development under the invert of a buried pipe was investigated using the mesh geometries shown in Figure 10. These lead to changes in circumferential stresses across the wall of the pipe at the invert, as illustrated in Figure 11. Once again, the stress distribution through the wall of the pipe deviates somewhat from a linear distribution, given the thickness of the elastic pipe structure (though in this case, the changing distributions of wall stress suggested that the neutral axis also appears to deviate from the midpoint of the wall).

In each of these cases, the effect of the void at the invert is to reverse the sign of the bending moments that occur at crown, invert and springlines. The removal of the soil foundation below the pipe is analogous to the reductions in earth loads that result when a rigid pipe is placed on a compressible foam foundation, or is shielded from overburden

stresses as a result of induced trench construction. The void below the invert leads to substantial reductions in the vertical stresses that acting across the structure, and eventually the void is large enough to lower the vertical stresses carried across the pipe below the horizontal stresses it is carrying. At this point, the sign of the bending moments changes. Perhaps the only very significant circumstance where this would be of concern is for pipes with elliptical reinforcement, which greatly lowers their ability to resist these moments of opposite sign (since the steel is placed on the assumption that radius of curvature increases at the crown and invert, and it decreases at the springlines).

For the invert void, it appears that smooth interface response provides more rapid changes in bending moments and greater stresses at the extreme fiber.

## 7 DISCUSSION AND CONCLUSIONS

This study demonstrates that the development of a void at the springline results in substantial increases in bending moments at crown, springlines and invert. Elastic analysis implies that voids stretching around a 90° arc at the springline acts to approximately double the bending moments and tensile stresses at the extreme fibers. An elastic-plastic finite element analysis considering backfill of low strength approximately triples the bending moments and tensile stresses. These analyses suggest that the erosion voids can certainly bring the pipe to a stability limit state. The longitudinal fractures that open along the springlines as a result of these moments could then enhance further erosion.

At first when erosion begins to form a void beside the pipe springline, the increases in bending moment are modest and they have little impact on the balance between load and resistance. However, once the void at the springlines grows to a point where stretches over an arc extending more than  $45^\circ$ , moments begin to accelerate more rapidly. If joint repair is able to arrest void growth at this level, the analyses imply that the system will remain stable. The analyses of larger voids imply that these systems are very likely to have resistance fall below demand.

The analysis of voids formed under the invert implies that substantial reductions in moment result, followed the development of moments of opposite sign. This is analogous to use of geofoam or other compressible materials under the pipe invert, or induced trench construction (a void over the crown). This would be of major concern for pipes with elliptical reinforcement, which depend on moments of specific sign at each of the critical locations (crown, invert and springlines).

All results presented here are based on theoretical calculations. Improvements to the predictive capacity of the analysis will depend on the development of relevant physical data. In particular, buried pipe tests featuring voids at the springline would be valuable to assess the accuracy of the finite element calculations and the definitions of critical geometrical (e.g. void shape) and material (soil strength) properties.

#### ACKNOWLEDGEMENTS

This work has been funded by a Discovery Grant awarded by Canada's Natural Sciences and Engineering Research Council. Dr Moore's position at Queen's University is funded by the Canada Research Chairs program.

## REFERENCES

1. Hoeg, K. 1968. Stresses against underground structural cylinders, *J. Soil Mechanics and Foundation Engineering*, ASCE, 94, (4), 833-858.
2. Katona, M.G. (1978), Analysis of long-span culverts by the finite element method, *Transportation Research Record*, Transportation Research Board, Washington D.C., 678, 59-66.
3. Moore, I.D., 1989. Elastic buckling of buried flexible tubes - a review of theory and experiment. *Journal of Geotechnical Engineering*, American Society of Civil Engineers, Vol. 115, No. 3, pp. 340-358.

4. Moore, I.D. and Hu, F., 1995. Response of high-density polyethylene pipe in hoop compression. Transportation Research Record 1514, Design and Performance of Underground Pipe, pp. 29-36.
5. Dhar, A.S., Moore, I.D. and McGrath, T.J. 2004. Two-Dimensional Analysis of Thermoplastic Culvert Deformations and Strains, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 130, No. 2, pp. 199-208.
6. Dhar, A.S. and Moore, I.D. 2001. Liner buckling in profiled polyethylene pipes, Geosynthetics International, Vol. 8, No. 4, pp. 303-326.
7. National Research Council Canada, 1998. Durability and Performance of Gravity Pipes: A State-of-the-Art Literature Review, Inst. for Research in Construction.
8. Gumbel, J., Spasojevic, A., and Mair, R. 2003, Centrifuge modelling of soil load transfer to flexible sewer liners, Proceedings ASCE Pipeline 2003 Conference, Baltimore, 11 pp.
9. Moore, I.D. 2001. Culverts and buried pipelines. Chapter 18 of the Geotechnical and Geoenvironmental Handbook, pp. 541-568, Edited by R.K.Rowe, Kluwer Academic Publishers.

10. Carter, J.P. and Balaam, N.P. (1980). AFENA-A general Finite Element Algorithm: user manual. School of Civil and Mining Engineering, University of Sydney, N.S.W. 2006, Australia.

Flexural rigidity $EI$	0.19876	MN.m
Hoop stiffness $EA$	1232	MN/m
Thickness $t$	44	mm
Diameter $D$	344	mm
Modulus $E$	28000	MPa
$I$	7.0987E-06	m <sup>4</sup> /m
Length $L$	1	
Max. vertical force $F$	0.023	MN/m
Crown Moment $M_{cr}$	0.001259	MN.m/m
Extreme fiber stress $\sigma$ at crown	3.9026	MPa
Maximum stress	4	MPa

Table 1 Calculation results of the maximum bending moment and maximum stress for  
Class III Reinforced Concrete pipe under parallel plate loading.

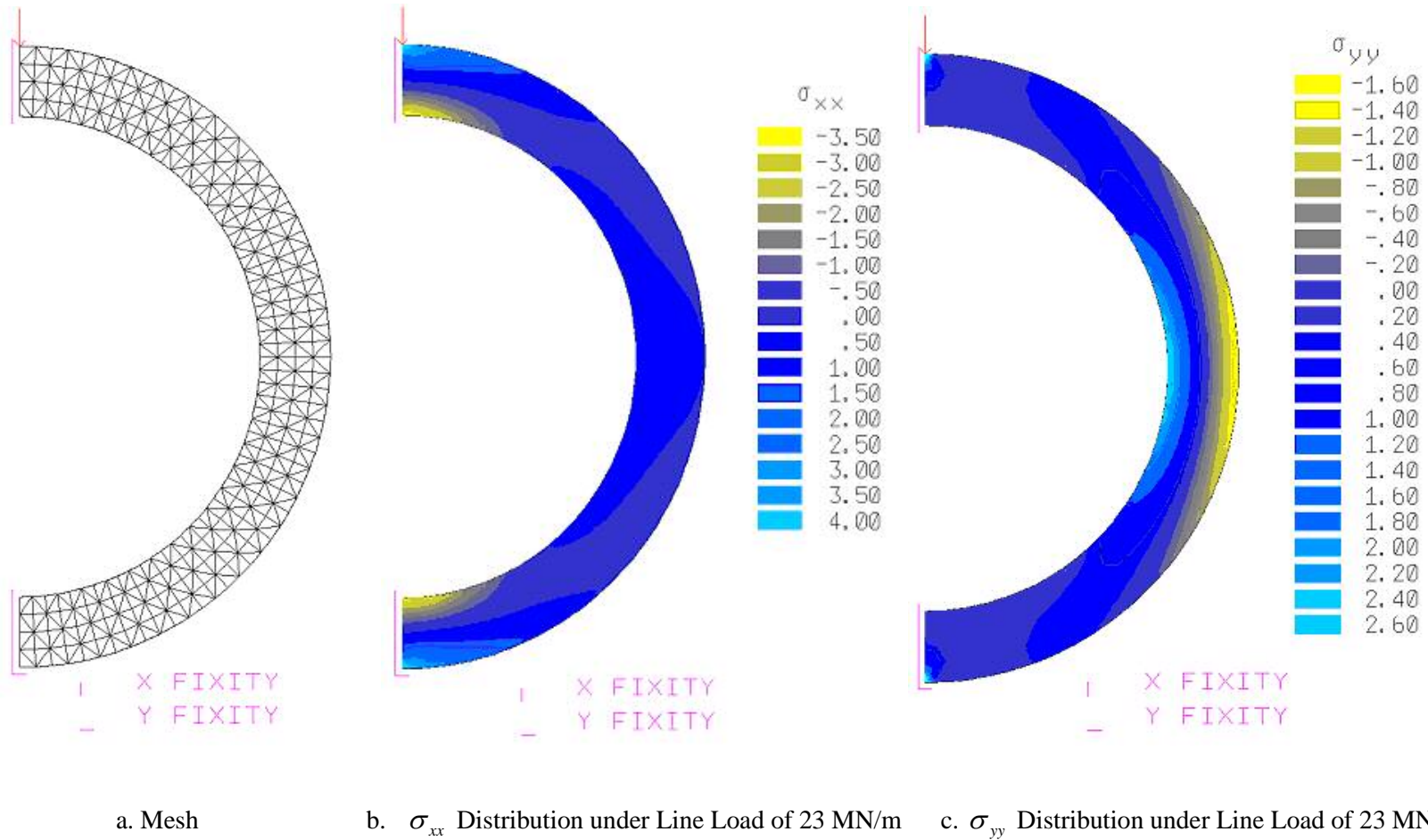


Figure 1 Finite Element Analysis of Rigid Pipe Alone (Tension Negative)

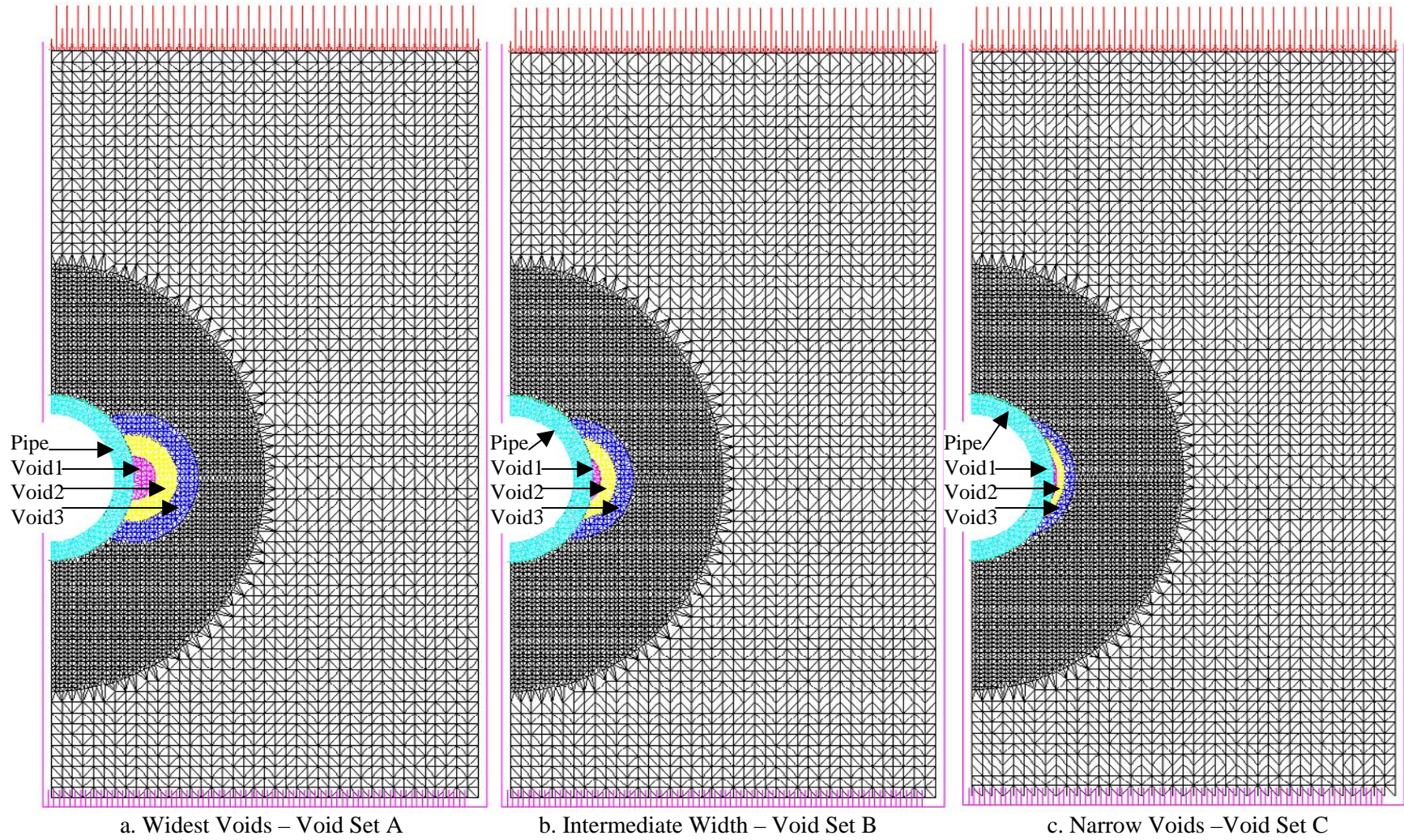


Figure 2 Finite Element Models of Buried RC Pipe with Voids at Springline

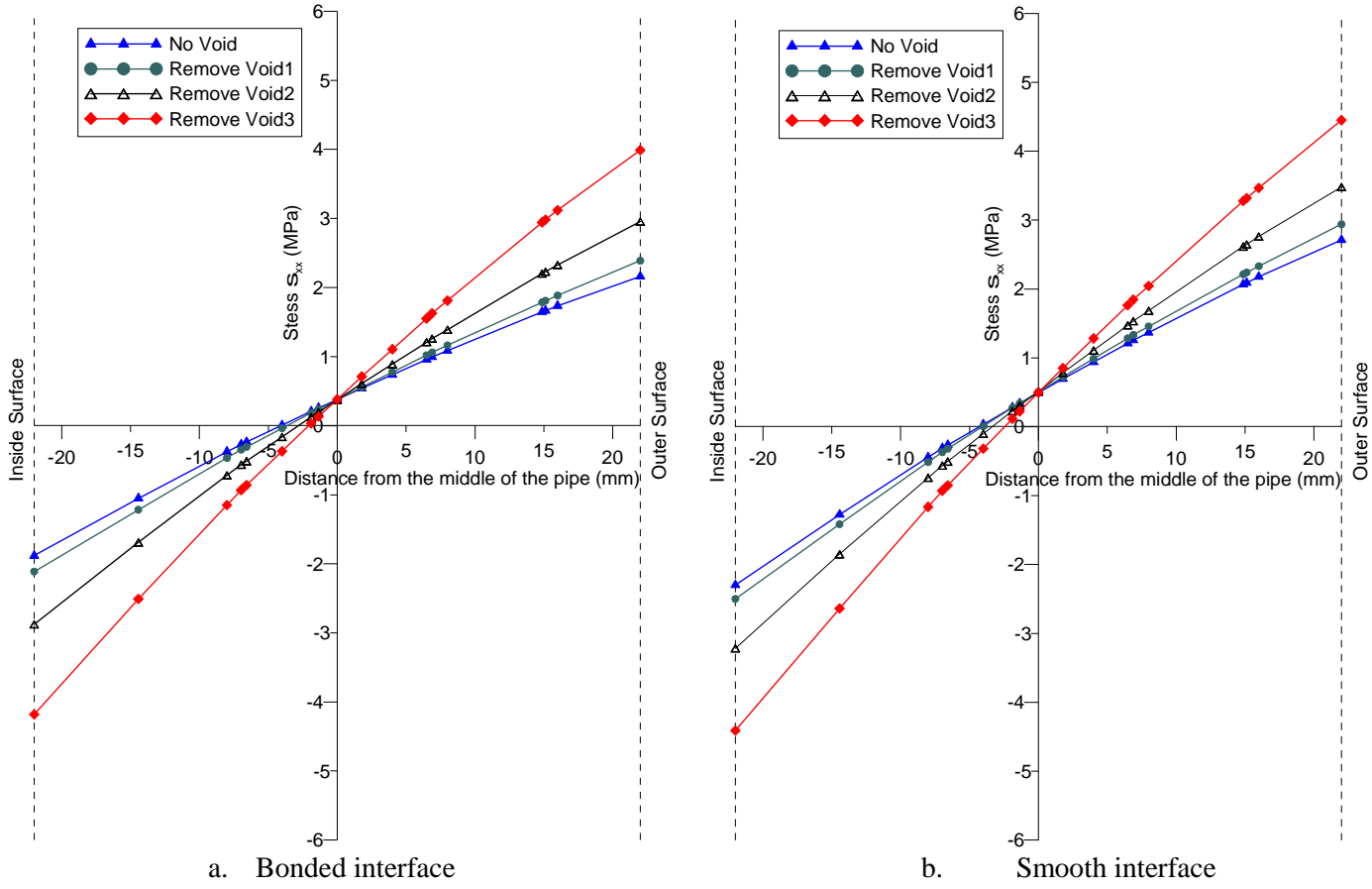
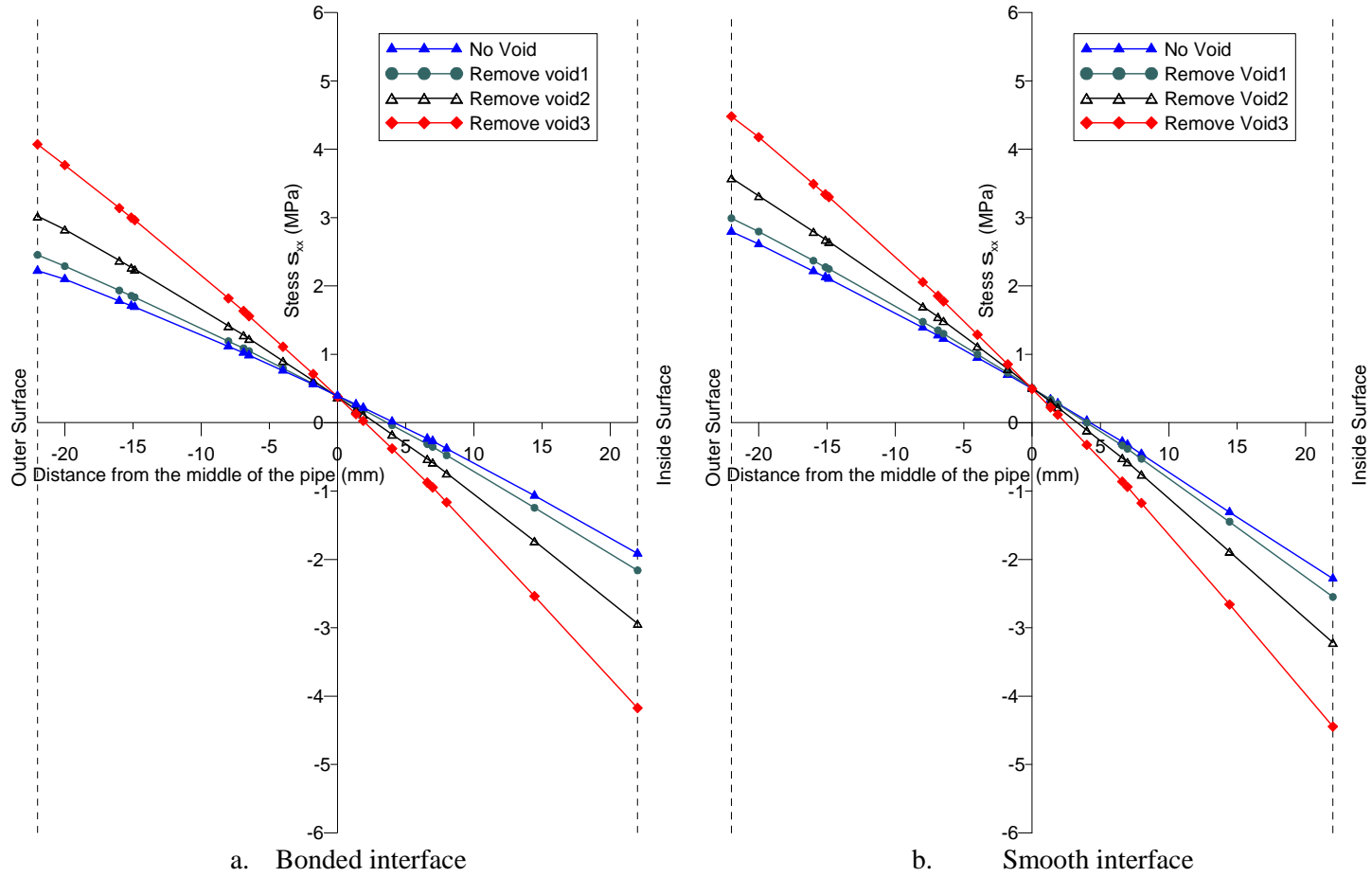


Figure 3 Stress  $\sigma_{xx}$  at Crown of the Rigid Pipe for Void Set A (Compression Positive)



a. Bonded interface  
 b. Smooth interface  
 Figure 4 Stress  $\sigma_{xx}$  at Invert of the Rigid Pipe Void Set A (Compression Positive)

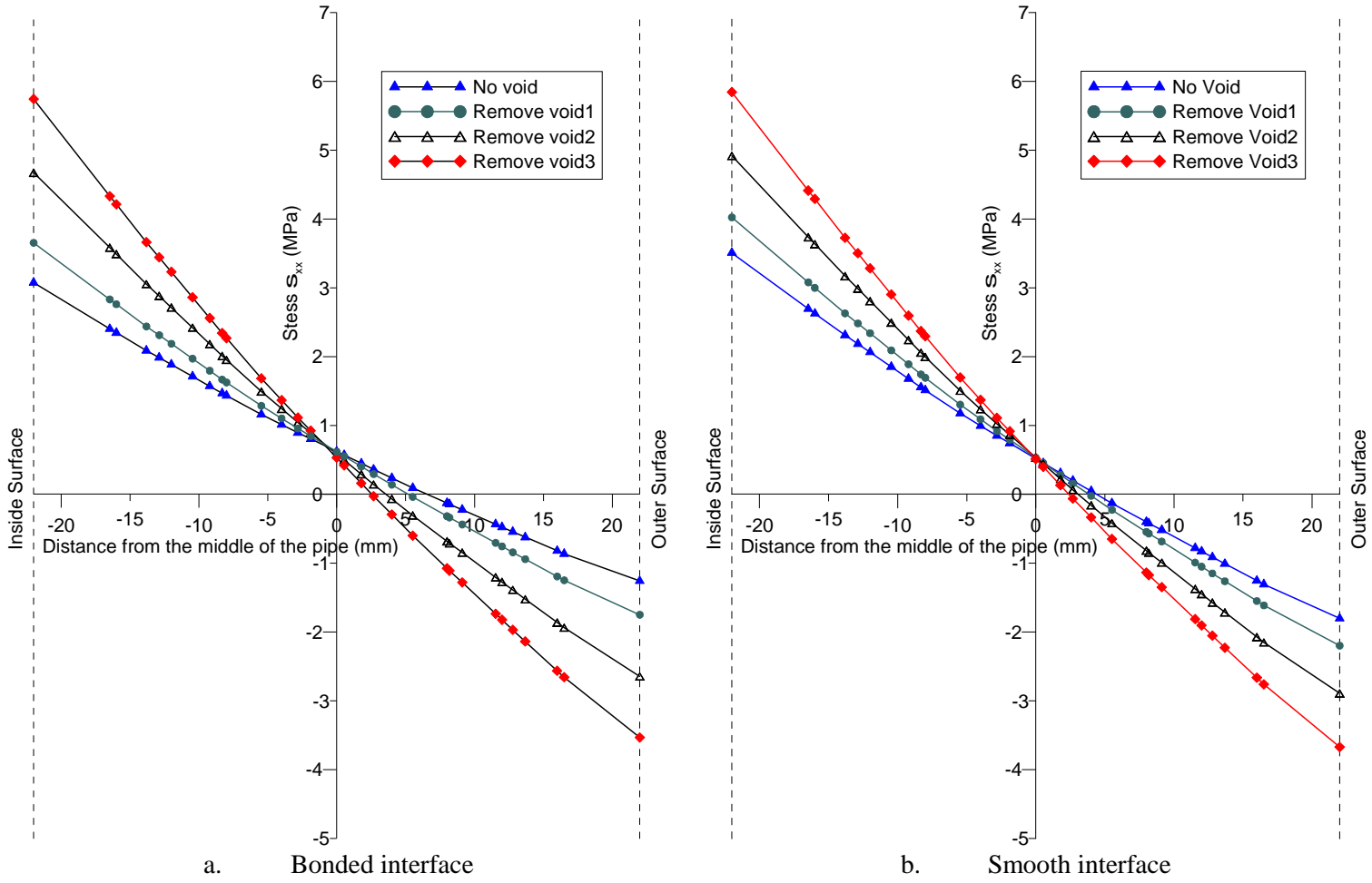


Figure 5 Stress  $\sigma_{yy}$  at Springline of the Rigid Pipe for Void Set A (Compression Positive)

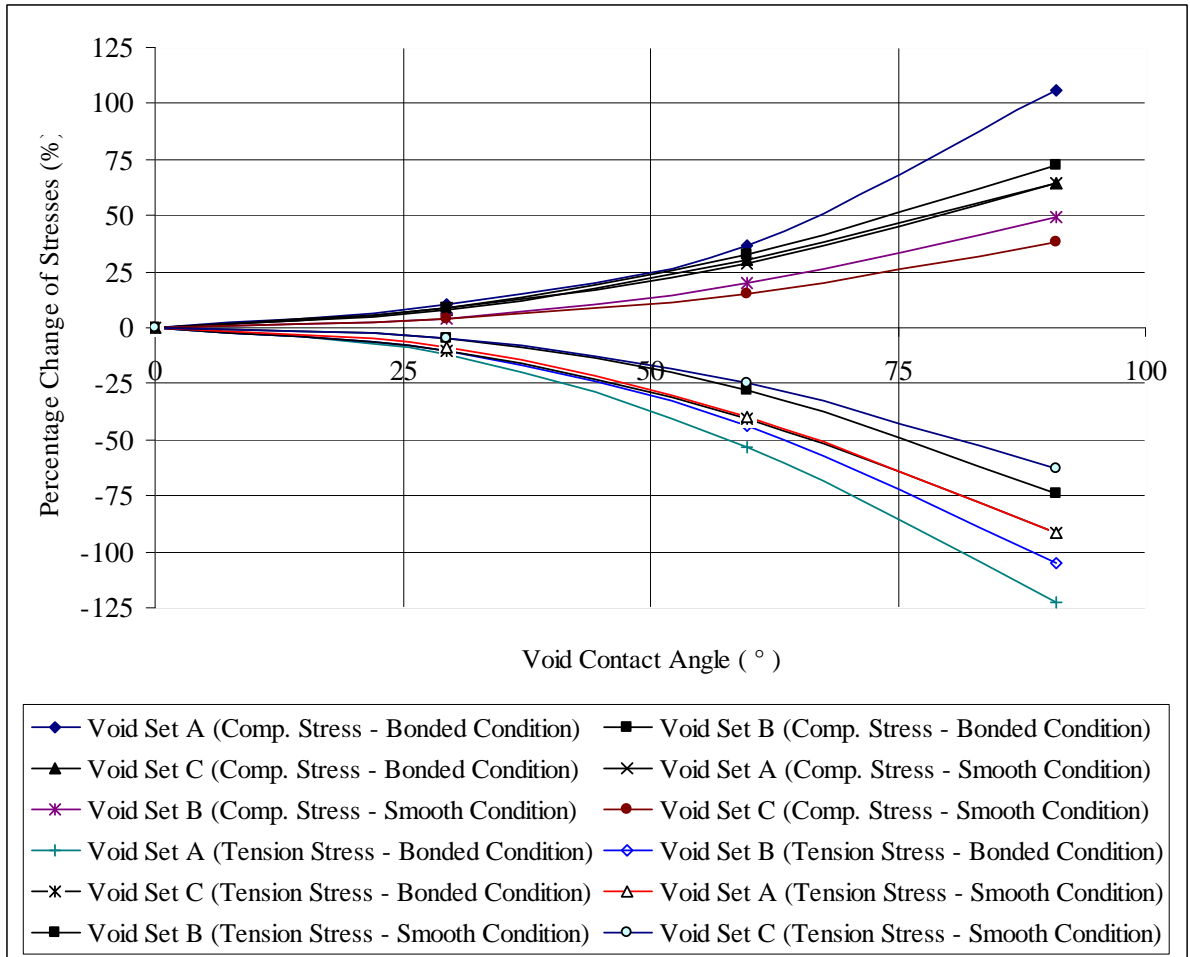


Figure 6 Elastic Analysis of Increases in Extreme Fiber Stresses at the Crown as a Function of Void Geometry and Interface Condition

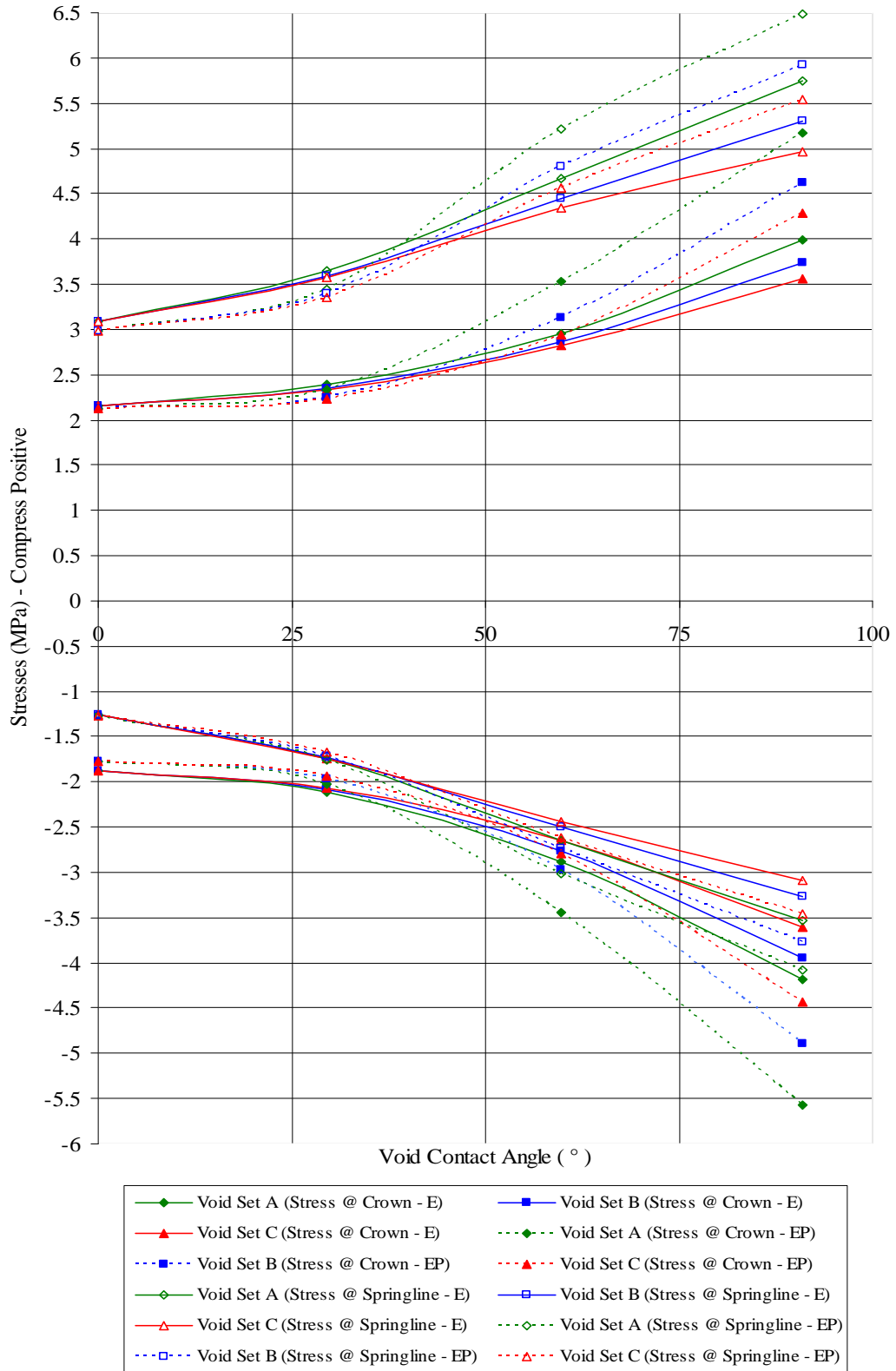


Figure 7 Extreme Fiber Stress Comparison (at the Crown and Springline) between Elastic Analysis and Elastic- Plastic Analysis for Bonded Interface (E – Elastic, EP – Elastic-Plastic)

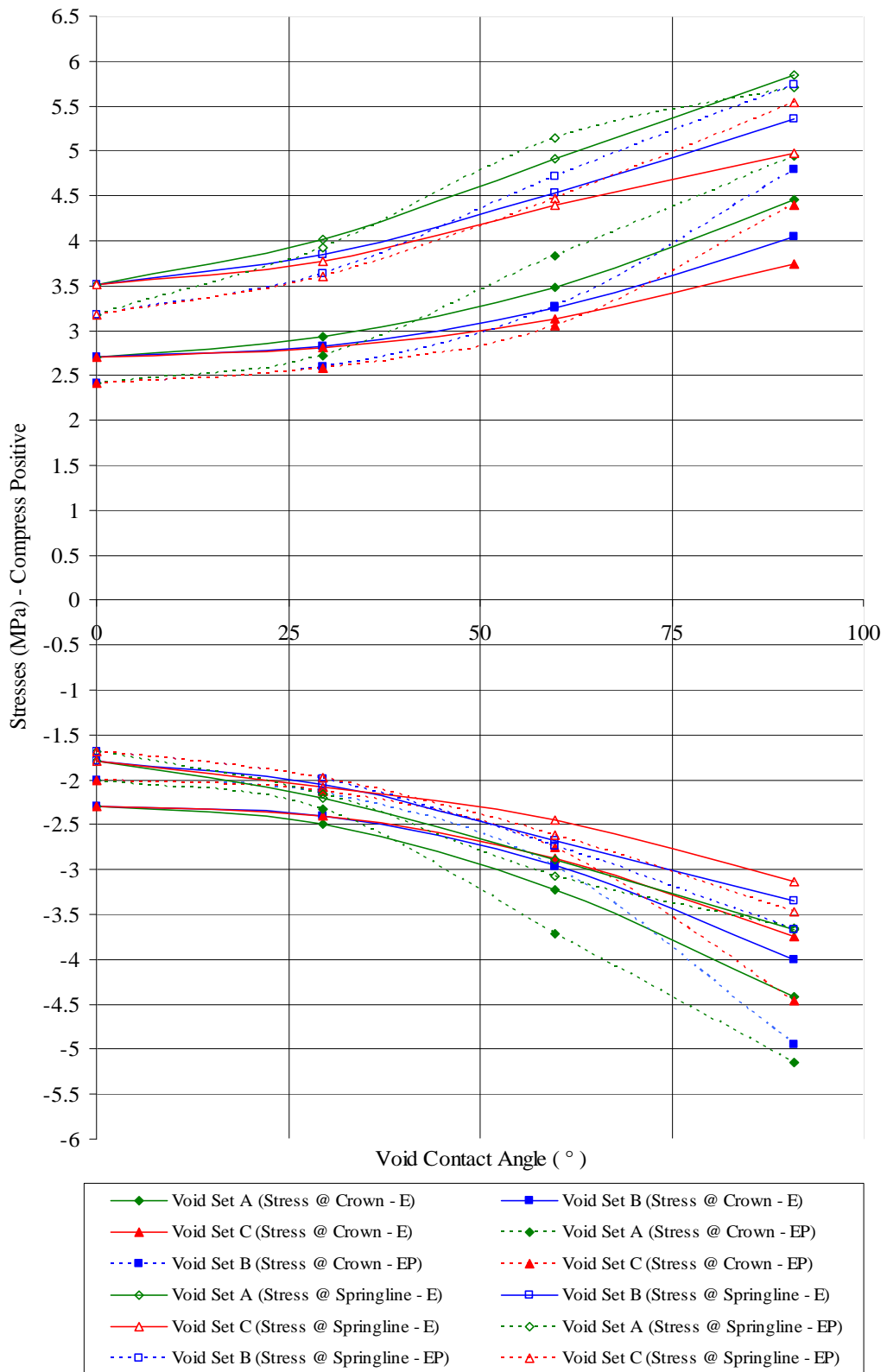


Figure 8 Extreme Fiber Stress Comparison (at the Crown and Springline) between Elastic Analysis and Elastic- Plastic Analysis for Smooth Interface (E – Elastic, EP – Elastic-Plastic)

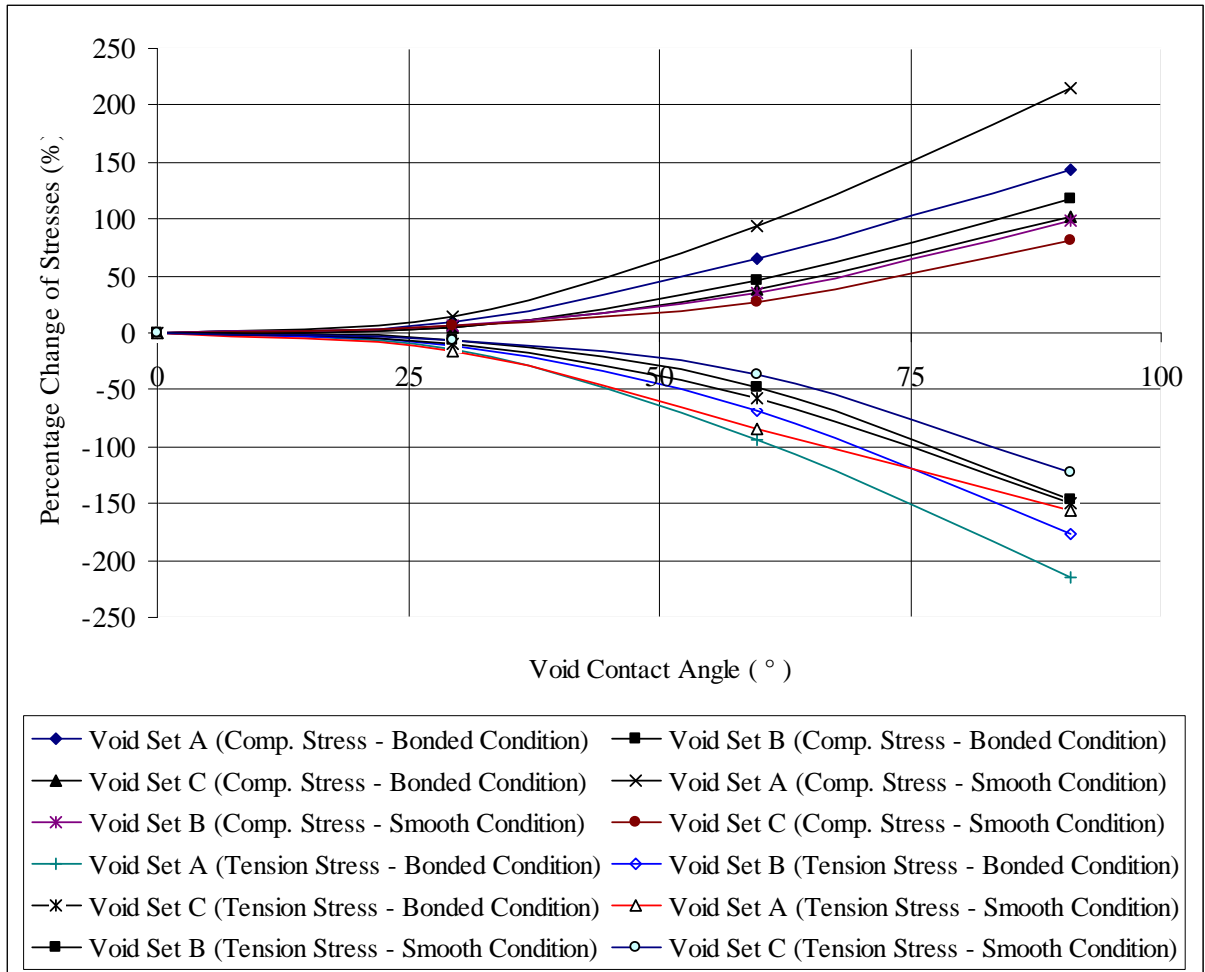


Figure 9 Elastic-Plastic Analysis of Increases in Extreme Fiber Stresses at the Crown as a Function of Void Geometry and Interface Condition

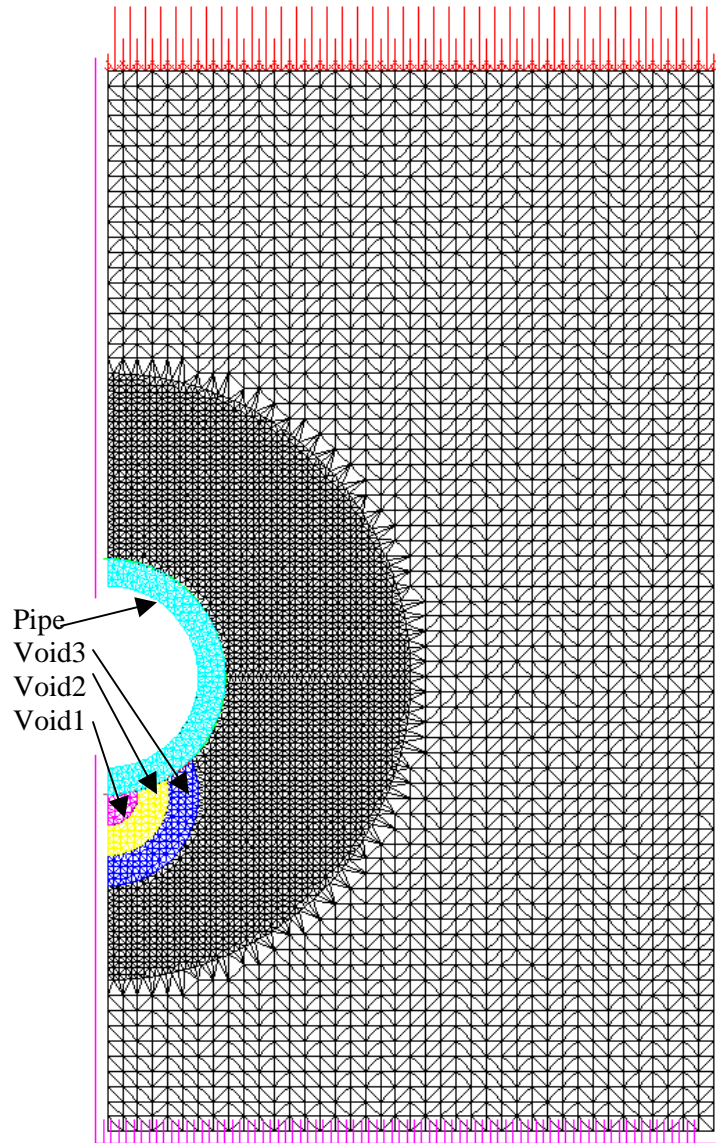
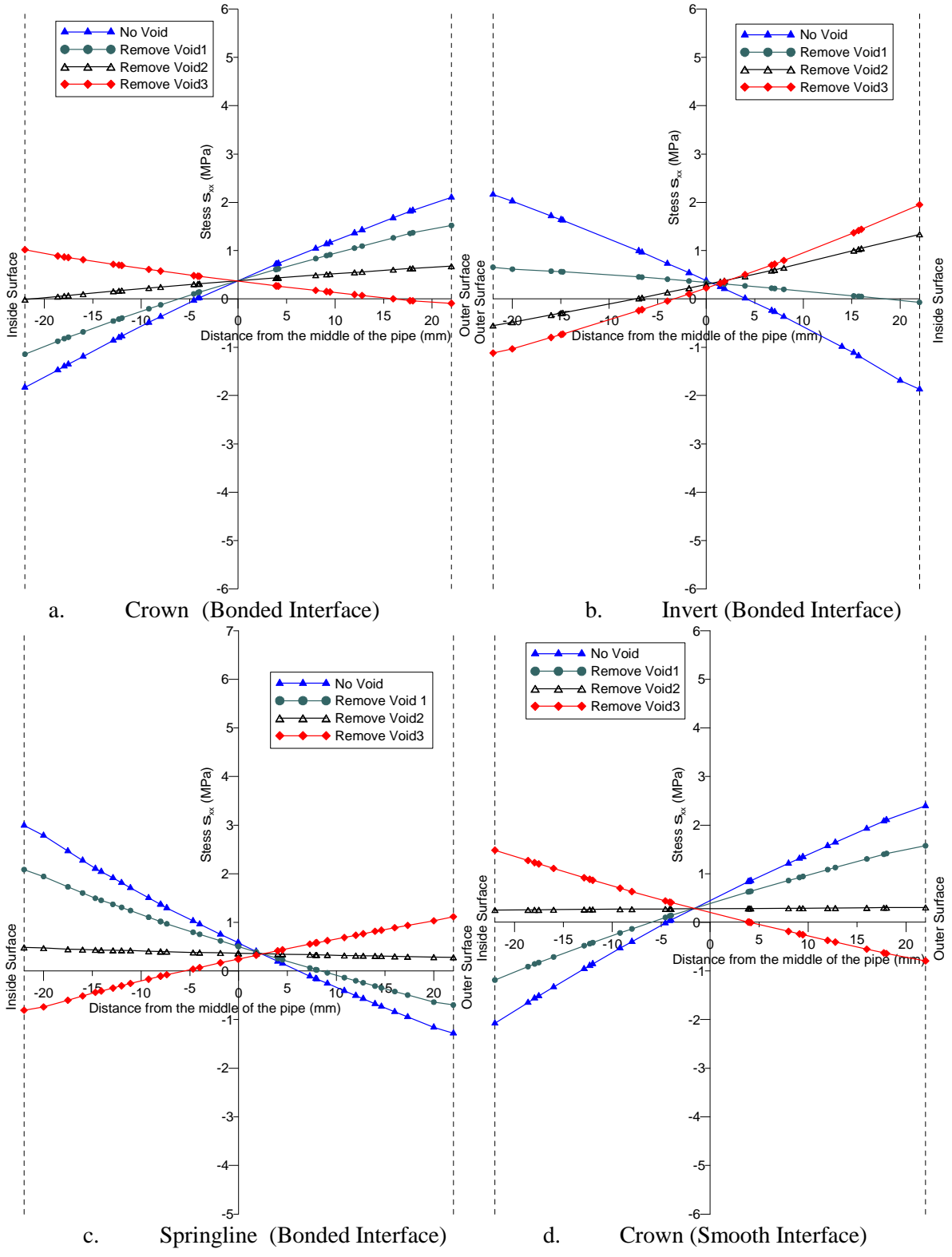
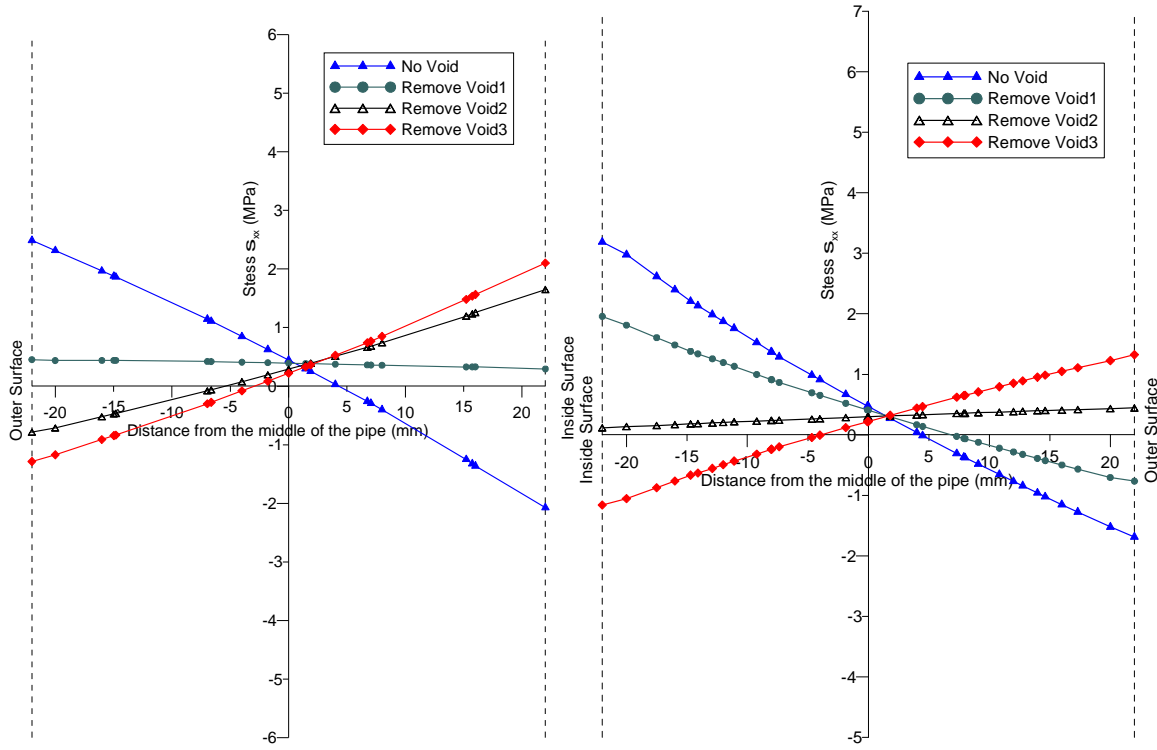


Figure 10 Mesh Defining Voids under Invert (Void Set D)





e. Invert (Smooth Interface)

f. Springline (Smooth Interface)

Figure 11 Circumferential Stress for Void at Invert (Void Set D)