

Constitutive Model for High Density Polyethylene to Capture Strain Reversal

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Abstract

Models available for estimating the rheological response of high density polyethylene (HDPE) are reviewed. A constitutive model combining elastic, viscoelastic, and viscoplastic components is proposed to calculate strain recovery and long term stress conditions in HDPE pipes pulled into place using trenchless construction methods. The ability of the new model to capture HDPE creep, relaxation, strain reversal, and strain accumulation and recovery during cyclic loading is examined by comparing simulations with experimental data. The new model is better able to provide response involving unloading and strain reversal. Temperature effects on the mechanical behaviour of HDPE are not considered in this paper.

Introduction

High density polyethylene is widely used in Trenchless installations of underground drainage and pressures pipes because it has low cost, light weight, and durability, and because it is relatively easy to weld pipe segments together in the field. Underground pipe installation or replacement using horizontal directional drilling and pipe bursting generally leads to complex axial load histories, where the high density polyethylene pipe is subject to cyclic loading as it is pulled into place. The long term implications of those installation loads have not been thoroughly investigated, partly because computational methods have not been available to estimate the axial pipe strains and stresses.

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The mechanical properties of high density polyethylene are highly time dependent and its response can differ significantly when subjected to different loading conditions with different rates and amplitudes. The mechanical behaviour of HDPE consists of an instantaneous elastic response, as well as viscoelastic (recoverable) and viscoplastic (irrecoverable) deformations that are time dependent. There is no well-defined yielding point beyond which permanent strains develop. Viscoplastic behaviour initiates at low stress levels.

Different constitutive models have been developed to simulate the mechanical behaviour of HDPE. These have primarily focused on HDPE under fixed stress or strain conditions. Chua (1986) used data from load tests on polyethylene pipes and proposed a time dependent relaxation modulus as follows:

$$E(t) = \frac{\sigma}{\varepsilon(t)} = 52.6 + 460 t^{0.097786} \quad (1)$$

where E is the modulus in MPa and t is time in hours. Hashash (1991) performed tests on corrugated HDPE pipes and proposed the following time dependent modulus:

$$E(t) = \frac{\sigma}{\varepsilon(t)} = 329 t^{0.0859} \quad (2)$$

Zhang and Moore (1997) carried out extensive testing on cylindrical polyethylene samples extracted from commercially available plain HDPE pipes. The results of creep tests at different levels of engineering stress were used to develop a nonlinear viscoelastic model. Zhang and Moore (1997) also developed a viscoplastic model using their data for HDPE pipe response at various constant engineering strain rates.

Most of the models have proven effective in calculations of HDPE response to monotonic loading. However, calculations of pipe response associated with unloading, strain reversal, and cyclic loading have been poor. Axial load histories of this kind are expected during horizontal directional drilling and pipe bursting since in both cases, the process of pulling pipes through the ground involves axial tensile force applied to the pipe that fluctuates significantly.

The authors (Chehab and Moore, 2004) reported on a linear viscoelastic-viscoplastic model to estimate the cyclic load response of HDPE. While the performance of that model was better than the alternatives, it still under-estimates the strain recovery and over-estimates the permanent strains. Further improvements were required, to ensure that the contribution to strain of the viscoplastic component of that mode was not exaggerated.

Uniaxial Viscoelastic-Viscoplastic Model

The mechanical behaviour of high density polyethylene includes elastic, viscoelastic and viscoplastic components. The model developed in this study combines, in series, an elastic sub-model, a viscoelastic (VE) sub-model, and a viscoplastic (VP) sub-model. The elastic sub-model is simply a spring that represents the instantaneous response of the material. The viscoelastic sub-model can be mechanically represented, as described by Moore and Hu (1996), by a combination of nine Kelvin elements in series. The viscoelastic constitutive relations can be written as:

$$\sigma(t) = E_0 \varepsilon_0 = E_i \varepsilon_i + \eta_i \dot{\varepsilon}_i \quad (3a)$$

$$\varepsilon_{ve}(t) = \sum_{i=1}^9 \varepsilon_i(t) \quad (3b)$$

where $\sigma(t)$ is the true applied stress, E_0 and ε_0 are the modulus of elasticity and strain of the elastic sub-model, and E_i , η_i , ε_i and $\dot{\varepsilon}_i$ are the elasticity, viscosity constant, strain, and strain rate of the i th Kelvin element. The elasticity and damping constants of the second to ninth Kelvin elements were chosen to be multiples of the elasticity and damping constants, respectively, of the first Kelvin element, as shown in Figure 1. Nonlinearity was introduced in the VE sub-model to capture changes in the magnitude and rate of strain recovery as strain magnitude increases. This is achieved by defining the damping constants as a function of plastic work, i.e.

$$\eta_i = \eta_{i0} \exp(-3W_{vp}) \quad (4a)$$

$$\text{where } W_{vp} = \int \sigma d\varepsilon_{vp} \quad (4b)$$

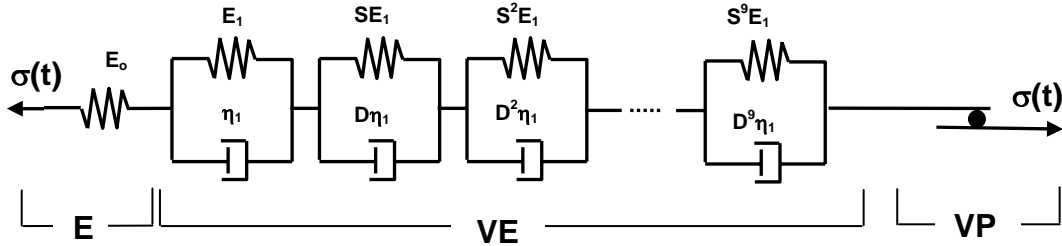


Figure 1: The VE-VP model proposed in the study

The viscoplastic sub-model was developed in a manner similar to the one proposed by Zhang and Moore (1997). It relates the plastic strain rates to both the stress level and the viscoplastic work done (to account for the effect of the loading history) as shown in the equations below:

$$\dot{\varepsilon}_{vp} = C \left(\frac{\sigma - \sigma_0}{X} \right)^n \quad (n \geq 1) \quad (5)$$

where $\dot{\epsilon}_{vp}$ is the viscoplastic strain rate, C is a scalar constant and σ_o is a threshold stress above which plastic strains initiate (this threshold stress could also be considered to be the yield stress). The state variable x is defined as:

$$x(W_{vp}, \dot{\epsilon}_{vp}) = \frac{1}{\alpha(\dot{\epsilon}_{vp}) + \sqrt{\beta/(\gamma + W_{vp})}} \quad (6)$$

where W_{vp} is the viscoplastic work and α , β and γ are model parameters.

The results of uniaxial compression tests at four constant engineering strain rates (Zhang and Moore, 1997) were used to develop the model. For the given stress histories, the instantaneous elastic strain ϵ_e and viscoelastic strain ϵ_{ve} are calculated using the elastic and viscoelastic sub-models featuring the following parameters (modified after Moore and Hu, 1996):

$$\begin{aligned} E_o = 1120 \text{ MPa} & \quad \} \quad \text{ELASTIC} \\ E_1 = 3615.5 \text{ MPa} & \\ \eta_{1o} = 43459.2 \text{ MPa.s} & \quad \text{VISCOELASTIC} \\ S = 0.845 \quad D = 5.0 & \end{aligned} \quad (7)$$

Parameters S and D are multipliers to determine the elasticity and damping for the second to ninth Kelvin elements, as shown in Figure 1. The viscoplastic strain component ϵ_{vp} is then calculated as:

$$\epsilon_{vp}(t) = \epsilon(t) - \epsilon_{ve}(t) - \epsilon_e(t) \quad (8)$$

for total strain $\epsilon(t)$ obtained from the experiments. Figure 2 illustrates the elastic, viscoelastic and viscoplastic strain components for one specific constant strain rate test.

The viscoplastic strain rates were calculated from the viscoplastic strains using central divided difference numerical differentiation. The viscoplastic work, given by Eq.4b, was calculated using Simpson's numerical integration. For each set of strain rate test results, two sets of the state variable x were calculated using Equations 5 and 6, starting with selected values of the parameters C , n , α , β and γ . Nonlinear curve fitting was performed to match the two sets as shown in Figure 3, with the parameters determined to be:

$$\begin{aligned} C = 0.01 & \quad n = 8 \\ \sigma_o = 5 \text{ MPa} & \\ \alpha = 0.01718 + 0.04581 \dot{\epsilon}_{vp}^{0.1984} & \\ \beta = 0.0158 \text{ MPa} & \\ \gamma = 0.015 \text{ MPa} & \end{aligned} \quad (9)$$

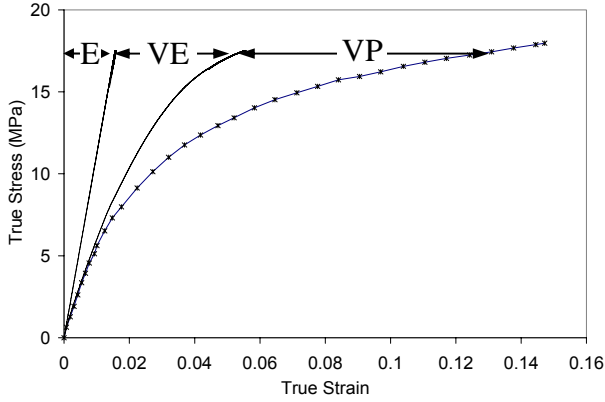


Figure 2: Elastic E, viscoelastic VE, and viscoplastic strain components.

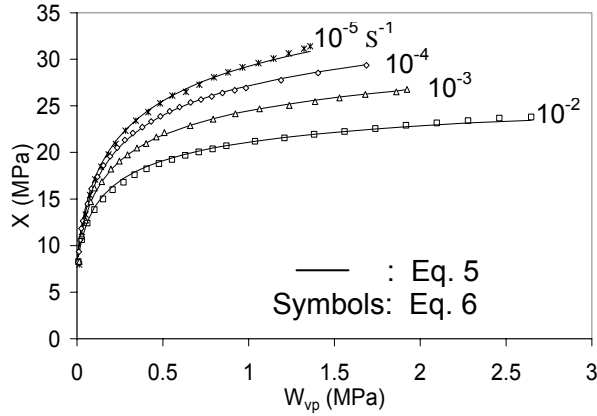


Figure 3: State variable X for different strain rate tests.

Figure 3 shows that very effective curve fitting is achieved for the four different strain rates.

Numerical Integration

For a test where axial displacements are controlled, the strain history is known and the stress history must be calculated. All model parameters are updated at the end of every time step. The viscosity constants in the VE sub-model (η_i) are updated using Eq.4a with the updated viscoplastic work W_{vp} (from Eq.4b). The viscoplastic sub-model parameter α is also updated using the last calculated viscoplastic strain rate (from Eq.9).

Using the updated parameters, the VE strain increment is calculated by solving Eq.3 using Laplace Transform techniques. The state variable X is calculated using Eq.6 with updated viscoplastic work and parameters. The VP strain increment is then calculated by multiplying the VP strain rate (from Eq.5) by the time increment Δt . Finally, the viscoelastic and viscoplastic strain increments are subtracted from the

total applied strain increment to obtain the elastic strain increment, which is multiplied by the elastic modulus E_0 to determine the resulting stress increment.

For a load control test, where the stress history is known and the strain history is desired, the stress increment is used to calculate the elastic, viscoelastic and viscoplastic strain increments using the continuously updated parameters. Individual components of the strain increment are then added to provide the total resulting strain increment.

Model Evaluation

The new model was used to calculate response to different loading conditions and these were compared with experimental results reported by Zhang and Moore (1997). Unless otherwise stated, the experimental results and the model calculations are expressed in terms of true stress and true strain, which are related to nominal stress and strain, respectively, by (Zhang, 1996):

$$\sigma = \sigma_{eng} \exp(-\epsilon) \quad (10a)$$

$$\epsilon = -\ln(1 - \epsilon_{eng}) \quad (10b)$$

Figure 4 shows the strain histories for creep tests at five different engineering stress levels. It can be seen that the model calculations agree well with the experimental results, but slightly under-estimates the short term creep recovery.

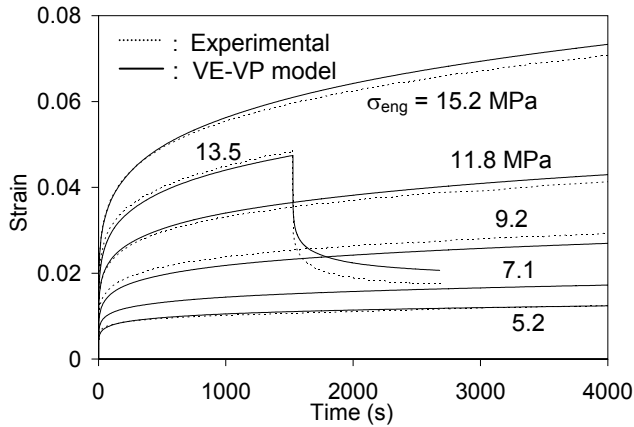


Figure 4: Test results and model simulation for creep tests with recovery allowed in one test.

Modeling cyclic behaviour is a challenge and none of the existing models is sufficiently accurate in calculating HDPE response to cyclic loading. For example, the nonlinear viscoelastic and the viscoplastic models developed by Zhang (1996) worked very well for most monotonic loading conditions, but poorly for unloading or cyclic conditions. The VE-VP model developed in this study was tested for these more challenging loading conditions as shown in the following figures.

Figure 5 shows the experimental results and model calculations for four constant engineering strain rate tests. In terms of loading, the model works very well except at high strains (over 12 %) where it under-estimates the stress. The model captures the initial unloading response very well down to a stress level of about 5 MPa, below which it tends to underestimate the recovery as unloading continues. The results and model calculation for a cyclic loading test are shown in Figure 6. Both the recovery and the hysteresis are slightly underestimated, but the general behaviour is good.

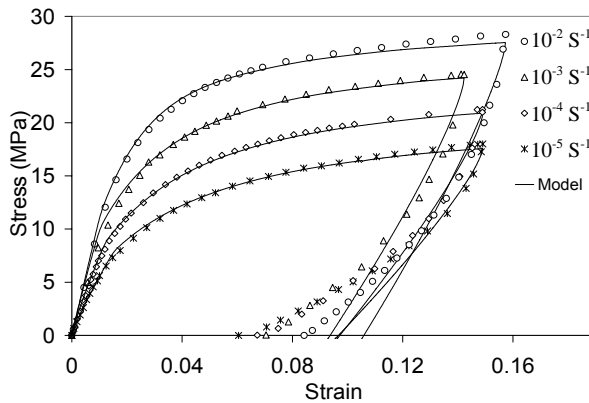


Figure 5: Model simulation for strain reversal with reversal rates equal to the straining rates.

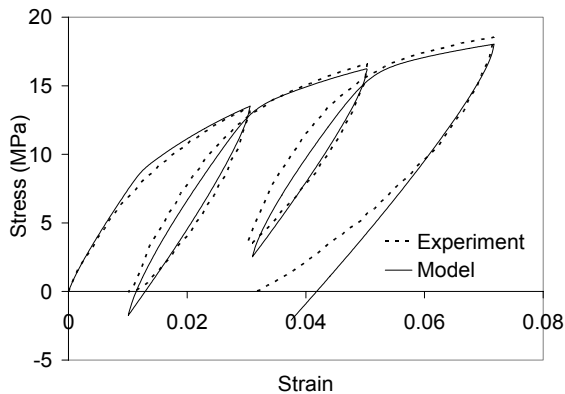


Figure 6: Cyclic loading, loading and unloading at strain rate of 10^{-4} s^{-1} .

Conclusion

A constitutive model that combines elastic, viscoelastic and viscoplastic sub-models was developed in this study. Damping in the viscoelastic sub-model is defined as a function of viscoplastic work (i.e. it is load history dependent). The parameters of the viscoplastic sub-model were determined by curve fitting a state variable using the results of four constant engineering strain rate tests.

The mechanical response of high density polyethylene is significantly influenced by the loading rate, stress level and load history. Although the model worked well for the cases presented in this paper, it should be examined with other, more complex, loading combinations and histories. Further data is being collected so that additional comparisons can be undertaken.

The uniaxial model developed in this study can readily be incorporated in finite element codes to study uniaxial pipe response during trenchless installation (horizontal directional drilling and pipe bursting). Established techniques can also be used to generalize the model to multi-axial stress conditions, so it can be incorporated into more general finite element algorithms.

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