Two-Dimensional Analyses of Thermoplastic Culvert Deformations and Strains

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Abstract: Tests have been performed in a biaxial pipe test cell to develop baseline information on profiled pipe behavior under biaxial loading. These include a lined corrugated high-density polyethylene pipe and a helically wound ribbed polyvinyl chloride pipe. Results of the tests are utilized to examine the effectiveness of the two-dimensional methods of buried pipe analysis. Calculations of pipe responses by the two-dimensional finite element method and a set of simplified design equations are compared with the measurements of pipe strains and deflections. The study reveals that the two-dimensional finite element analysis can effectively be used to calculate pipe deflections and circumferential strains. The simplified equations appear suitable as design tools for standard buried thermoplastic pipe installations. Janbu's nonlinear soil model with Mohr–Coulomb plasticity provided an effective simulation of the nonlinear soil behavior A study of pipes with low stiffness soil support under the haunches shows that this leads to strain concentrations in the pipe walls near that zone. Higher values of empirical strain factor, D_f, are estimated to include this strain concentration during design.

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Introduction

A number of limit states are important to the design of buried thermoplastic pipes. Pipe deflections have traditionally been the prime focus of attention when designing to maintain the serviceability of a culvert structure, with deflection limits between 5 and 7.5% specified in various codes of practice. Pipe deformations are also associated with the development of circumferential strains in the pipe wall, strains that can induce other limit states. Geometrical instability can arise in the form of local buckling (Moore and Laidlaw 1997; Dhar and Moore 2001), which can develop in various slender elements of the pipe profile. Local strain of sufficient magnitude also represents a material limit, with ductile failure associated with large compressive stresses, and cracking possible in elements under the action of sustained tensions (Moore and Hu 1995, 1996).

A collaborative research project through the National Cooperative Highway Research Program is directed towards developing two types of limit states design methods for buried thermoplastic pipes. First, a procedure is being developed based on simplified design equations, suitable for hand calculation or inclusion in a spreadsheet. Vertical diameter decrease Δ_v is obtained using a modified form of the Spangler (1941) and Burns and Richard (1964) equations developed by McGrath (1998a)

$$\frac{\Delta_v}{D}=\left(\frac{q_v}{\dfrac{EA}{R}+0.57M_s}\right)+\left(\frac{D_l K q_v}{\dfrac{EI}{R^3}+0.061M_s}\right) \tag{1}$$

The first term represents the hoop compression, while the second is the conventional term quantifying bending deformations. The constrained modulus M_s is used instead of the semiempirical soil stiffness parameter E' employed by Spangler (1941), McGrath (1998b). Other parameters are defined as follows: D=pipe diameter; q_v=overburden pressure at springline; E=pipe material modulus; R=radius of the centroid of the pipe section; M_s=one dimensional soil modulus; K=bedding coefficient; D_l=deflection lag factor; A=area of the cross section; and I=moment of inertia. Peak circumferential bending strain is then expressed as a function of displacement using empirical "shape" factor D_f (Watkins et al. 1973)

$$\epsilon_b=D_f\left(\frac{c}{R}\right)\left(\frac{\Delta_v}{D}\right) \tag{2}$$

where c=distance to the extreme fiber from the neutral surface within the pipe wall.

Secondly, two-dimensional finite element analysis is widely available, and this tool is being used with increasing frequency to design specific higher cost culvert installations. A "comprehensive design method" employing two-dimensional finite element analysis is therefore being developed, so that culvert designers can use this analysis to improve design economy when warranted by the project cost.

To date, no study on polyvinyl chloride (PVC) or high-density polyethylene (HDPE) pipes has reported comparisons between measurements of local wall strain and deflection, and two-dimensional finite element calculations. Tests were therefore per-

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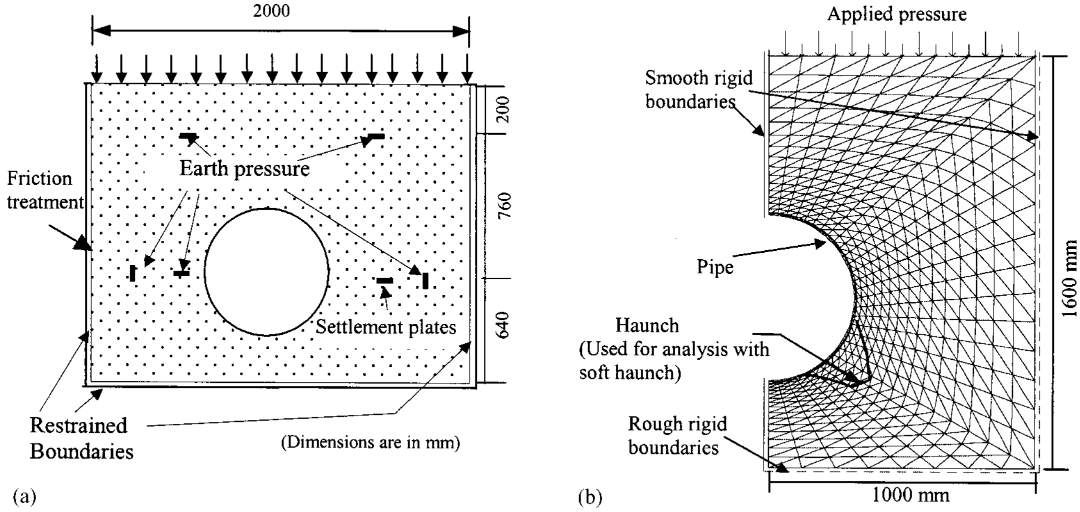

Fig. 1. Test cell and finite element mesh

formed on lined corrugated HDPE pipe with annular (axisymmetric) geometry, and helically ribbed PVC pipe. A second HDPE pipe was also tested; results were similar and are not presented here in the interests of brevity. The laboratory tests were performed in the biaxial pipe test cell at the University of Western Ontario, a cell developed to model the biaxial field stresses experienced by pipes deeply buried under embankments (Brachman et al. 2000). Comparisons are made of deflection and wall strains, to evaluate the effectiveness of the computer analysis and the simplified procedure for calculations of pipe behavior under deep burial. The computer analysis is then used to examine one situation involving non-uniform soil support—studying the magnitudes of deflection and local surface strain for both the HDPE and PVC pipes backfilled with zones of low stiffness soil beneath the haunches. The study concludes with recommendations regarding the strengths and limitations of two-dimensional finite element analysis, and aspects of the analysis that need to be addressed to obtain useful calculations of pipe deflections and circumferential strains. Conclusions are also made regarding the magnitude of local surface strains, and values of strain factor D_f for use in simplified design calculations of circumferential bending strains at the extreme fibers.

Test Instrumentation and Measurements

Test Cell Arrangement

The biaxial test cell at the Univ. of Western Ontario is a high strength steel box with dimensions 2 m×2 m in plan and 1.6 m in height. Details of the design of the cell are reported by Brachman et al. (2000), and the arrangement of the pipe and instrumentation for these tests are as shown in Fig. 1(a). Pipes of about 600 mm in diameter were placed horizontally equidistant from the sidewalls. This leaves backfill of 650 mm width on both sides of the pipe. Design of the cell (Brachman et al. 2000) included a special sidewall treatment (Tognon et al. 1999) to minimize the sidewall friction, ensuring that soil response approaches the one-dimensional settlement condition once the lateral distance exceeds one pipe diameter (as demonstrated by the finite element analysis of Dhar, 2002).

Soil bedding below the pipe is 340 mm thick. Poorly graded sand (uniformity coefficient, $C_u = 3.4$; coefficient of curvature, $C_c = 1.1$) was used as the backfill material. The backfill soil was compacted to a density of about 1,625 kg/m^3, which is 85% of the maximum standard Proctor density. Earth pressure cells were used to measure the vertical and horizontal stresses at the springline and at 200 mm below the top surface of the soil. Two settlement plates were positioned at the springline level to monitor the vertical soil movements. An air bladder was used to apply uniform pressures on top of the soil, Fig. 1(a).

Tests were conducted in pressure increments of 25 kPa, with each increment allowed to remain for 20 min, a sufficient period of time to permit the cell pressure to stabilize. Thus the average rate of loading was 0.021 kPa/s. Loading was continued for about 6 h until limit states, such as local buckling, were observed and a vertical pressure of 500 kPa was reached.

Description of Profiles

Fig. 2 shows the two pipe profiles considered in this study. The first one is a twin walled (lined corrugated) annular HDPE pipe with internal diameter of 610 mm. The pipe has 101 mm pitch and 55.2 mm corrugation depth. The second structure is a PVC pipe of 605 mm internal diameter and helically ribbed profile. The angle of helix of the rib is about 9° with average clear spacing between the ribs of 35.8 mm.

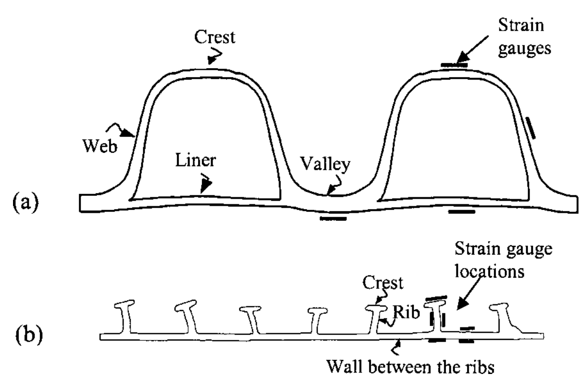

Fig. 2. Pipe profiles and strain gage locations: (a) lined corrugated high-density polyethylene pipe; (b) ribbed polyvinyl choride profile

Pipe Instrumentation

Linear variable differential transducers were used to measure the changes in horizontal and vertical diameters of the pipes. Wall strains on different components of the profile were measured using resistance strain gages. Biaxial strain gauges were used for annular profiles to measure the circumferential and the axial strain. Strain gauge rosettes were used for the profile with helically wound ribs. Strain measurements were obtained at the crown, invert and springline (on both sides) of the pipe. Instrumentation was placed at two sections to check reproducibility of the test results and also to ensure that the data is collected in the event that some of the gauges are inoperative. The locations of the strain gauges on the profiles are shown in Fig. 2.

Finite Element Modeling of Test Cell

The Spangler (1941) equation has been used extensively in design to calculate the deflection of buried flexible pipes, and the closed form solutions of Burns and Richard (1964) and Hoeg (1968) are also being used to calculate pipe response under deep burial (e.g., Moore 2001). The continuum solutions are based on a number of idealizations, including the modeling of the soil as an infinite, homogeneous, isotropic, and linear elastic material. The two-dimensional finite element method is used when evaluations of the buried pipe response need to consider the materially nonlinear soil behavior, or where pipe installation in nonuniform ground is expected to have a significant influence on the pipe response (Chua and Lytton 1987; Katona 1988; Hashash and Selig 1990). Duncan's hyperbolic soil model (Duncan and Chang 1970), and its extension by Selig (1988) have generally been utilized to characterize the nonlinear elastic behavior of the soil. Time dependent behavior of thermoplastic material has been represented in most of these two-dimensional (2D) analyses using secant elastic modulus selected to characterize stress divided by strain for the polymer over the time period under consideration (generally focusing specifically on the short-term or long-term values), e.g., Katona (1988); Hashash and Selig (1990); and Moore (1995). Brown and Lytton (1984) adopted a simple power law formulation to account for the reduction of HDPE modulus with time, and Moore and Zhang (1998) have developed and used linear viscoelastic, nonlinear viscoelastic, and viscoplastic models for HDPE. These time-dependent material models are not yet available to most pipe designers conducting finite element analysis, so the present study employs 2D finite element analysis based on secant elastic modulus for the polymer (this is discussed in more detail in a subsequent section).

Small strain (geometrically linear) finite element analysis has been employed to study the interaction of thermoplastic pipe with the backfill soil in the test cell. The vertical cross section of the test cell being analyzed is shown in Fig. 1(b). Two noded beam–column elements were used to represent the pipe. The elements are defined along the neutral surface of the pipe wall. Area (A) and moment of inertia (I) of the sections were calculated by explicit integration of the actual profile geometries, and were expressed per unit length along the pipe axis. Six noded plane strain continuum elements were used to model the surrounding backfill.

Using symmetry, only half of the test cell and the pipe within need to be analyzed. Sixty structural elements were used around one half of the pipe circumference. A smooth rigid boundary was used to idealize the line of symmetry, where horizontal displacements were prevented and the crown and the invert of the pipe were restrained against rotation. A smooth horizontally restrained boundary was used for the sidewalls (the conventional approach since restraint of vertical degrees of freedom will severely reduce the earth pressures that reach the pipe). The base of the test cell was not lubricated, and nodal points along the bottom boundary were fixed in both horizontal and vertical directions.

Model Parameters for Thermoplastic Pipes

Parameters required for the beam–column elements are axial stiffness (EA) and the flexural stiffness (EI), where E is the elastic modulus of the material. The moduli for thermoplastic materials (HDPE and PVC) are dependent on time effects. As discussed earlier, elastic modeling using secant modulus is employed. American Association of State Highway and Transportation Officials (AASHTO 1998) provides guidelines for short-term and long-term values of modulus for HDPE as 760 and 152 MPa, respectively. The values recommended for PVC are 3,030 and 1,090 MPa.

Previous analysis by the authors of a biaxial test with similar duration and rate of loading showed that the pipe response was successfully calculated using either a nonlinear time dependent pipe material model or linear elasticity based on secant modulus (Dhar and Moore 2000a). Thus modulus of elasticity, E for the high-density polyethylene is taken as 450 MPa, representing a secant value for 6 h, the duration of the experiments, based on the data and viscoplastic modeling of Zhang and Moore (1997). Poisson's ratio ν of 0.46 was used for the HDPE. Modulus E, and Poisson's ratio ν, used for the PVC pipe are 2,760 MPa and 0.3, respectively (Sargand et al. 1995). Hoop stiffness (EA) and flexural rigidity (EI) of the sections were estimated as 4,200 N/mm and 1.40×10^6 N mm^2/mm, respectively, for the profiled HDPE pipe, and 19,480.1 N/mm and 1.14×10^6 N mm^2/mm, respectively, for the ribbed PVC pipe.

Assessment of Soil Parameters

Stress–strain properties of the soil were determined from the measurements of soil stresses and soil deformation in each of the tests (using the settlement plate and stress cell located in the bottom right hand corner of the cell). Finite element analysis reveals that settlements in this region occur under essentially one-

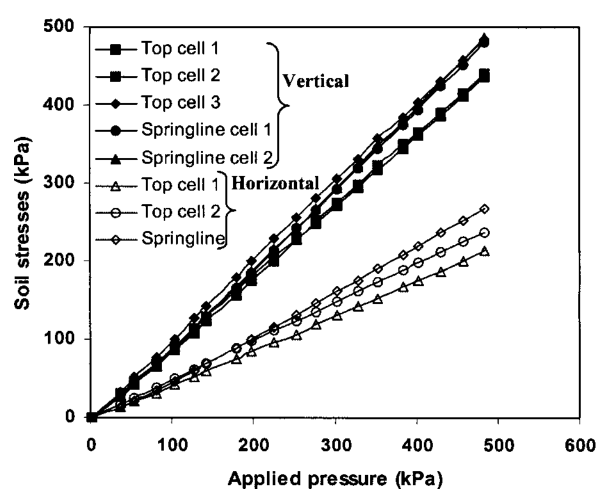

Fig. 3. Soil stresses measured in polyvinyl chloride pipe test

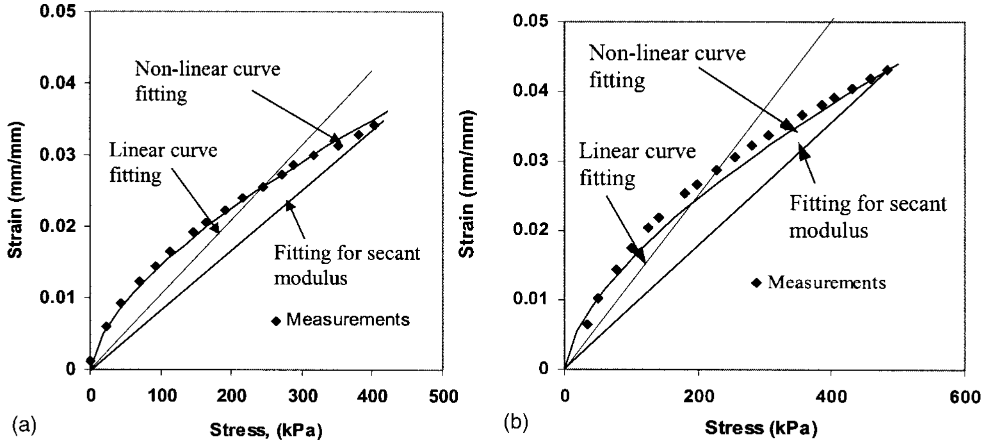

Fig. 4. Stress–strain relation of soil in high-density polyethylene pipe test (a) high-density polyethylene pipe; (b) polyvinyl chloride pipe

dimensional conditions (almost zero lateral strain). Coefficient of lateral earth pressure at rest, K_0, of 0.5 was obtained from the measurements of horizontal and vertical stresses for both tests. Typical soil stress measurements in one of the tests are shown in Fig. 3. Poisson's ratio was calculated from the K_0 value as $\nu = K_0/(1+K_0)=0.33$ to ensure that lateral earth pressures for zero lateral strain like those measured in the test are calculated by the elasto-plastic finite element analysis.

The stress–strain relations for the soil appeared to be nonlinear and stress dependent (Fig. 4). Vertical strain is plotted against vertical stress in these figures. The nonlinear model of Janbu (1963) was used to characterize this nonlinear stress-dependent soil behavior. Tangent modulus in this power law model is defined as $E=K\sigma^n$, where σ is the mean stress. Parameters (K,n) for the model were estimated from curve fitting to the experimental stress–strain relations. Resulting soil parameters are $n=0.37$ and $K=12.5\,(\mathrm{MPa})^n$ for the HDPE pipe test soil, and $n=0.37$ and $K=11.7\,(\mathrm{MPa})^n$ for the soil in the PVC pipe test.

In each case, values of equivalent linear modulus have also been estimated for specific stress levels (Table 1) since these are used later in the study to compare solutions based on linear and nonlinear soil models. Linear and nonlinear curve fits for the tests are shown in Figs. 4(a and b), where stress is plotted on the horizontal axis and the strain on the vertical axis in the figures. An elasto-plastic model based on the Mohr–Coulomb failure criterion was used to characterize shear failure in the soil. Angle of internal friction for the backfill soil, ϕ, of 30° was estimated based on the soil density. An associated flow rule was used to define the plastic deformation of the soil, that is a dilation angle of 30°. An analysis was also performed using a smaller dilation angle, 13°, a typical value for granular material (Skempton 1984), but this did not significantly change the finite element results (results changed by less than 1%).

It is worthwhile emphasizing that all soil parameters were obtained independently without reference to the measured pipe response.

Analysis of Test Results

Since the 2D models idealize the problem as plane strain, only the circumferential strain can be calculated. Measurements of the deflection and the circumferential strains are compared in this section with those obtained using the finite element method and the simplified design equations.

Comparison of Pipe Deflections

Finite Element Analysis

Measured deflections are compared with the finite element calculation for the two tests in Figs. 5 and 6 respectively. Analysis using nonlinear soil modeling shows good agreement with the measured deflections in both cases. It reveals that accurate modelling of the soil is important in the analysis of pipe–soil interaction since the soil stiffness largely controls the deformation of buried flexible pipe. The Janbu model appears to effectively model the soil response and the pipe within it; discrepancies in the first data point from each test likely resulted because this low overburden stress was not sufficient to mobilize slip along the lubricated side walls of the cell.

Elasto-plastic finite element analyses undertaken with constant (secant) Young's modulus soil model appear to provide results

Table 1. Moduli and Changes in Vertical Diameter at 100 and 400 kPa

Stress (kPa)	Test pipe	Expt.	FE	Secant modulus (MPa) M	Secant modulus (MPa) E	Average modulus (MPa) M	Average modulus (MPa) E	Deflection using secant modulus (mm) a	Deflection using secant modulus (mm) b	Deflection using average modulus (mm) a	Deflection using average modulus (mm) b
100	HDPE	−16.8	−16.9	6.8	5.1	4.6	3.4	−15.6	−16.1	−19.3	−20.0
	PVC	−13.1	−13.1	5.7	5.5	3.9	3.7	−14.2	−12.9	−14.7	−13.4
400	HDPE	−47.2	−48.1	11.5	9.6	7.7	6.5	−42.2	−44.6	−48.3	−50.4
	PVC	−34.6	−34.7	10.3	8.7	6.9	5.9	−34.4	−31.7	−39.7	−36.3

[a]Simplified method (McGrath 1998a).

[b]Continuum theory (Hoeg 1968): bonded interface.

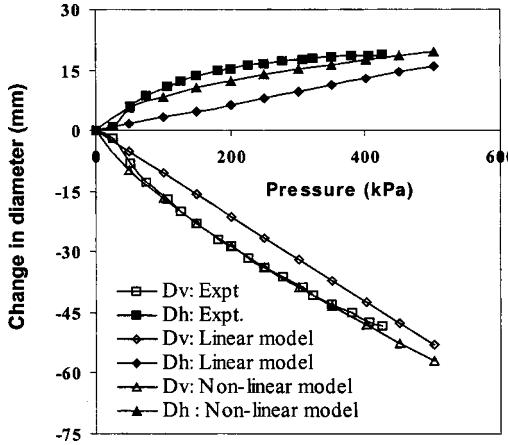

Fig. 5. Comparison of pipe deflection for high-density polyethylene pipe

similar to those obtained using Janbu elasticity, choosing the constant modulus as an average calculated over the 0–500 kPa stress range. This appears to effectively capture the accumulated effect of the confinement pressure on soil modulus, permitting pipe response to be effectively estimated at that peak stress level.

Simplified Methods

Moore (2001) describes use of the two-dimensional elastic continuum theory (Hoeg 1968) for design of buried pipes, showing how continuum theory can provide a unified design approach that can be applied to metal, concrete, and polymer pipes. However, the continuum theory equations are more complicated than those which can be developed to undertake design for only one class of product, thermoplastic pipes in this case, and that continuum approach cannot currently assess the influence of variable ground support (such as the loose, low stiffness soil typically placed under the pipe haunches). McGrath (1998a) proposed the simplified design equation presented earlier [Eq. (1)] based on the continuum approach (Burns and Richard 1964) to calculate the vertical deflection of flexible thermoplastic pipes. The equation is expressed in a form similar to the Iowa equation (Spangler 1941),

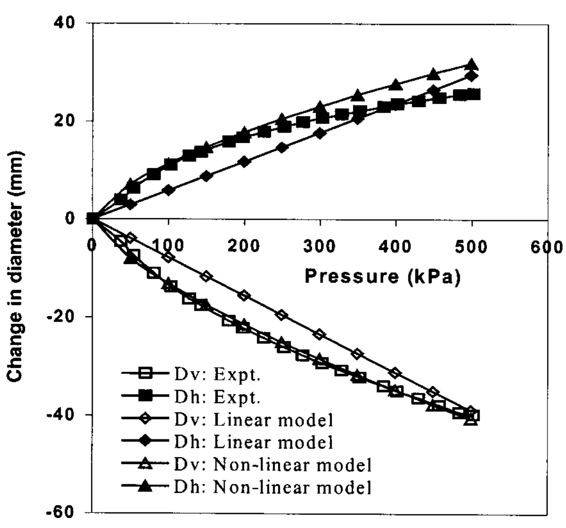

Fig. 6. Comparison of pipe deflection for polyvinyl chloride pipe

but incorporates contributions from both hoop compression and bending of the pipe–soil system.

The simplified design method (McGrath 1998a) is evaluated with the measurements and the finite element analysis in this section. A bedding factor of 0.083 is used in the simplified equation, corresponding to 180° bedding, since backfill in the test cell was placed with care to provide good support under the pipe (Spangler and Handy 1973). The constrained modulus, M_s is selected, corresponding to either the secant value or the average value, the latter obtained using a line of best fit to the nonlinear stress-strain relations, passing through the origin, Fig. 4. Constrained moduli and the corresponding Young's moduli at values of 100 and 400 kPa of applied stress are summarized in Table 1.

Calculations of pipe deflections are also summarized in Table 1, for various calculation methods and two vertical earth pressures, 100 and 400 kPa. Table 1 shows that continuum theory gives higher deflection for lined corrugated HDPE pipe and lower deflection for ribbed PVC pipe compared to McGrath's equation. This difference between the two different profiles implies that the structural stiffness of the pipe influences McGrath's simplified design calculation. The ribbed PVC pipe possesses higher hoop stiffness and lower bending stiffness compared to the corrugated HDPE pipe, implying that the simplified equation provides a more conservative estimate of pipe deflection for the pipe with high hoop stiffness.

Calculated deflections obtained using the finite element method at 100 and 400 kPa are also included in Table 1. Comparison of measurements with the calculated deflections indicates that the finite element method provides the best calculations of pipe deflection, likely due to its modeling of soil nonlinearity. The simplified equation renders best estimates when average backfill modulus is employed. The simplified method with secant soil modulus gives somewhat unconservative estimates of deflection. The effect of changing geometry may occur at higher stress and can be included in the finite element analysis (Dhar and Moore 2000b), but is not captured by the simplified procedure.

Pipe Strains

Simple beam theory has been used to calculate the strains from the values of thrust N and moment M obtained from the finite element analyses. The finite element method these stress resultants at the Gauss (numerical integration) points. Circumferential strain on a fiber located at a distance Y from the neutral axis of the section is given by

$$\epsilon=\frac{N}{EA}+\frac{MY}{EI} \tag{3}$$

Eq. (3) is based on the assumption that strain distribution is linear across the profile, so that all fibers located at the same distance from the neutral axis are modeled as responding with the same strain. Linear distribution of strain along the profile depth was measured during the experiments, except on the liner at the springline of the lined corrugated pipe (shown in Fig. 7 at overburden pressure of 200 kPa). Solid marks on Fig. 7(a) represent the strains on the liner with solid circle at the springline and solid triangle at the crown of the pipe. Circumferential strain on the liner is influenced by local bending, a three-dimensional response within the profiled pipe wall and therefore not included in two-dimensional theory (Moore and Hu 1995, 1996). However, Dhar and Moore (2000b) demonstrated that the two-dimensional theory could be used to calculate strains on the elements not subjected to local bending. Strains on two extreme fibers of each of the two

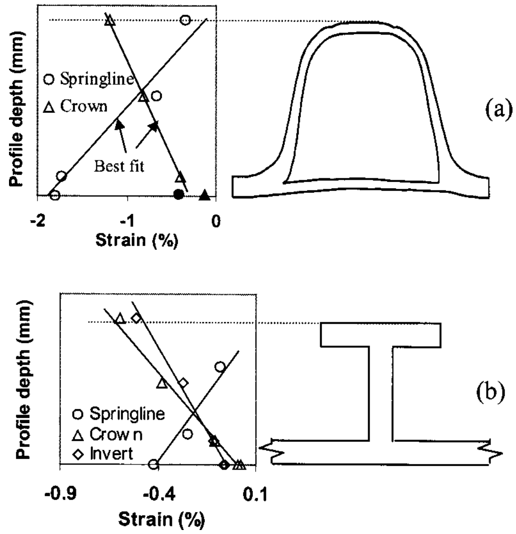

Fig. 7. Distribution of profile circumferential strains: (a) lined corrugated pipe; (b) ribbed polyvinyl chloride pipe

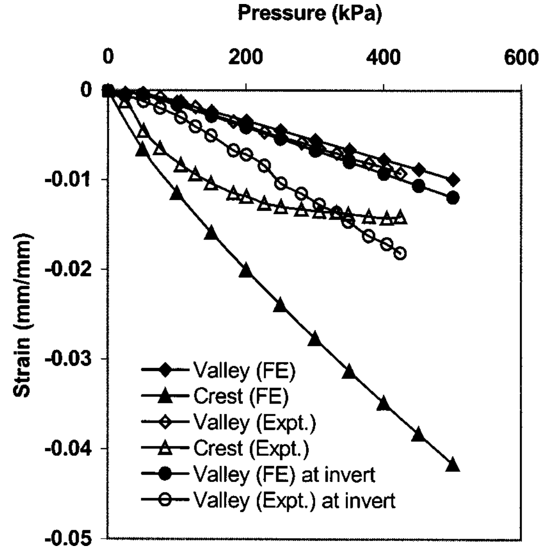

Fig. 9. Circumferential strains at crown and invert (high-density polyethylene pipe)

sections are examined here, the corrugation valley and corrugation crest of the HDPE pipe, and the inner wall and outer extremity of the ribs for the PVC structure.

Comparisons for circumferential strains at the springline, crown and invert of the HDPE pipe are shown in Figs. 8 and 9, which indicate general agreement between the measured strain and the calculated strain obtained from the 2D finite element analyses. Circumferential strains calculated at the springline (Fig. 8) are similar to the measured values, but are overestimated by about 12% on both the valley and the crest. At the crown, the calculated strain matches the measured strain (Fig. 9), except that the measured strain on the crest is a nonlinear function of overburden pressure, and stabilizes to a maximum value of 1.4%. Local buckling is believed to have occurred on the crest of the element at that strain. No strain reading was obtained for crest strain at the invert of the pipe. Measured and calculated strains on the valley are in accord initially (Fig. 9), and deviate at higher overburden stress, perhaps due to changes in the effect of nonuniform soil support near the invert.

For the PVC pipe examined, measurements of strains were made at the inner walls, rib web, and the crest of the profile [Fig. 2(b)]. Pipe internal strain under the rib and midway between the ribs was essentially the same at all the locations (crown, springline, and invert), implying that local bending is not important for this PVC profile.

Comparisons of measured and estimated strains in the PVC pipe also support the use of two-dimensional finite element modeling. Figs. 10–12 show general agreement between the strain calculations and the measurements. Strains on the inner surface and the web at the crown are almost coincident. On the crest, the calculated values initially follow the measured values, but deviate at higher stress levels. At the springline, strains on the inner wall are overestimated. Strain at the springline might have been reduced on the inner wall and increased on the crest due to insufficient compaction of soil below the pipe. Finite element estimation of strains are much higher at the invert, perhaps due to local nonuniformity of the soil in that region. Moreover, the trends of the measured and the calculated strains are similar. Effects of different haunch support on the pipe response are studied in the

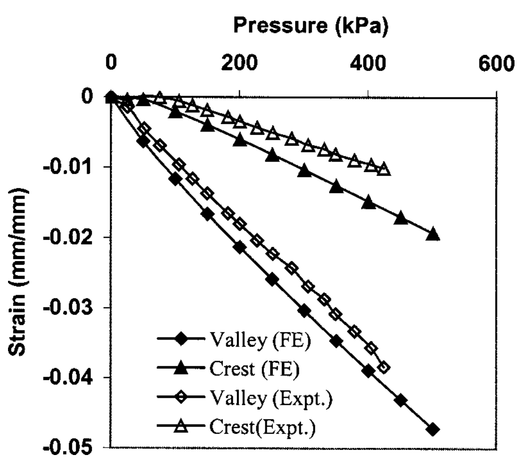

Fig. 8. Springline circumferential strains in high-density polyethylene pipe

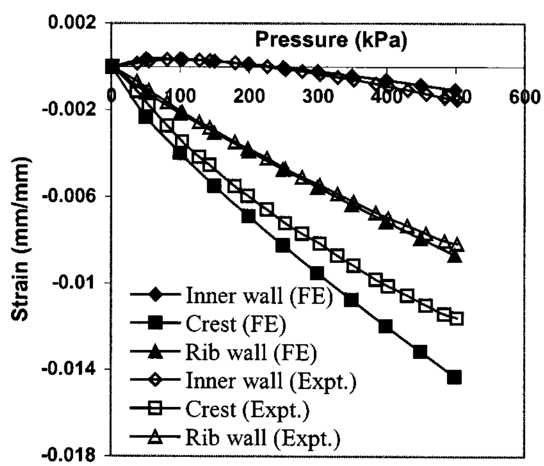

Fig. 10. Comparison of circumferential strain at crown of polyvinyl chloride pipe

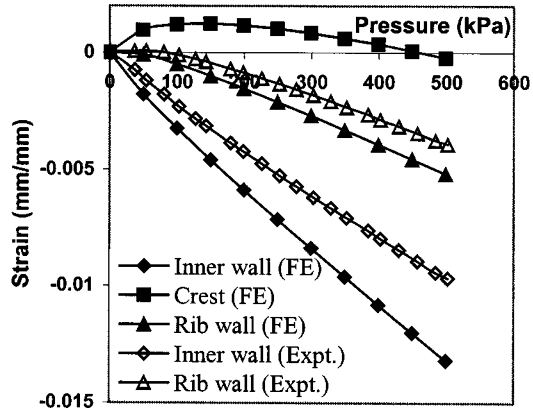

Fig. 11. Springline circumferential strain (polyvinyl chloride pipe)

next section of the paper, where these discrepancies are discussed in more detail. Nevertheless, the two-dimensional finite element method appears to provide reasonable estimates for this helically wound ribbed PVC pipe.

Strain on the inner surface (under or midway between the ribs) is found to be tensile at low levels of overburden stress as a result of the negative bending moments (compressive outward) at the crown and invert (Figs. 10 and 12). These strains revert to compressive values at higher stress levels where hoop thrust appears to govern the strains. Due to similar effects, the crest is subjected to tension at the springline (Fig. 11), but the strains remain tensile at that location up to much higher levels of vertical pressure.

Strain Factor for Peak Bending Strain

Watkins et al. (1973) proposed the simplified design equation [Eq. (2)] for peak bending strain, relating pipe deflection to strain through an empirical strain factor D_f. For bending of a circular incompressible ring that produces an elliptical deformed shape, the factor is 3 (Roark 1943). For compressible buried pipes, D_f may also depend on the stiffness of the pipe relative to the soil. To examine the factors for flexible pipes, values of the strain factors for the tests considered are calculated from the bending strains and the bending deflection, $(D_v - D_h)/2$, given by the finite element method. Analyses in the previous sections indicate that for uniform soil support, the finite element method provides a conservative estimate of the circumferential strain on the profiles of the pipes. The factors for both of the annular lined corrugated HDPE pipe and the helically wound ribbed PVC pipe were close to 3, the value for elliptical deformation.

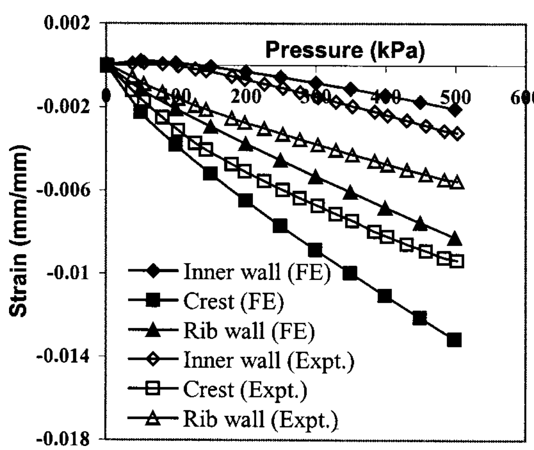

Fig. 12. Invert circumferential strains (polyvinyl chloride pipe)

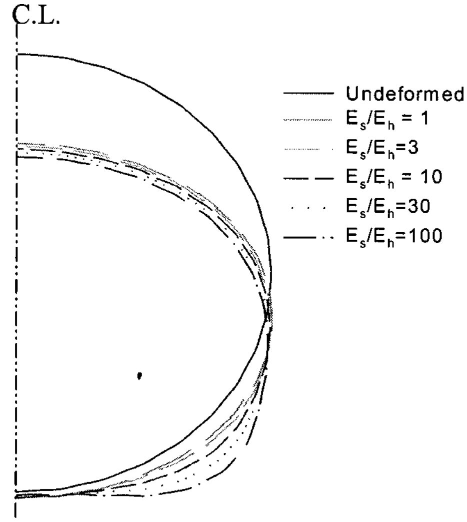

Fig. 13. Deformed shapes for high-density polyethylene pipe with different ratios of backfill to haunch soil modulus (deflection×5)

Study of Poor Haunch Support

Overall performance of buried pipe is strongly influenced by the behavior of the backfill material. Dense backfill generally provides better soil support to the thermoplastic pipe, but it is usually difficult to achieve uniform compaction around the pipe using conventional compaction methods, particularly when working in a pipe trench of limited width, where the soil below the pipe haunch cannot be accessed easily. Rogers et al. (1996) demonstrated that poor quality haunch support can produce odd-shaped displacement patterns and strain concentrations in the pipe. A brief study is presented here to examine the potential effects of poor haunch support on the pipe behavior.

The finite element method, successful in simulating the behavior of the two test pipes, is used here to investigate the influence of poor haunch support on both these structures. Fig. 1(b) shows the zone of low stiffness haunch soil considered in the analysis, a geometry consistent with that considered by McGrath et al. (1999), but with the side of the low stiffness zone extended up almost vertically towards the springline. Values of elastic modulus for this zone are reduced relative to the surrounding soil. Analysis is performed with constant elastic moduli and Mohr–Coulomb plasticity to account for shear failure. Modulus of the backfill soil was taken as 20 MPa, the value recommended by McGrath et al. (1999) for 70 kPa of overburden pressure, corresponding to overburden pressures from a moderate height (3.5 m) embankment constructed from dense granular soil. McGrath et al. (1999) proposed design values of backfill soil modulus for three general classes of soil (SW, ML, and CL) at different levels of vertical stress. The pipe material moduli used in the study were the same as those used earlier (i.e., 450 and 2,760 MPa for the HDPE and PVC, respectively).

HDPE pipe deformations are illustrated in Fig. 13 for different ratios of haunch soil modulus E_h to backfill modulus E_s, $E_s/E_h = 1$ to 100. The invert of the pipe flattens when the stiff-

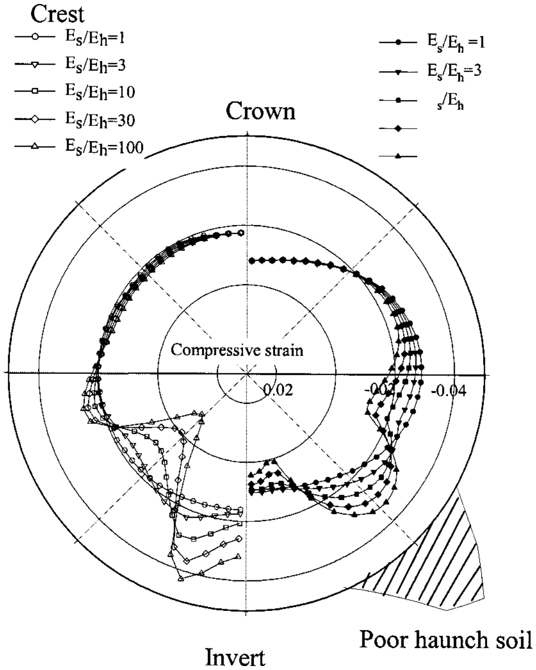

Fig. 14. Distribution of circumferential strains on high-density polyethylene pipe

ness of the soil at the haunch is reduced. This is largely consistent with the ‘‘inverted heart shape’’ deformation described by Rogers (1988), a shape characterized by flattening of the pipe at the invert. Rogers et al. (1996) indicate that inverted heart shape deformation occurs when the soil under the haunches is poor while that above it is of good quality. Fig. 13 implies that the vertical diameter change may be affected if the reductions in soil stiffness adjacent to the haunch are substantial.

Distributions of circumferential strains on the extreme fibers (on the valley or the inner wall and the crest) of the sections are plotted in Figs. 14 and 15, for the HDPE and PVC pipes, respectively. The figures illustrate that the low stiffness haunch soil

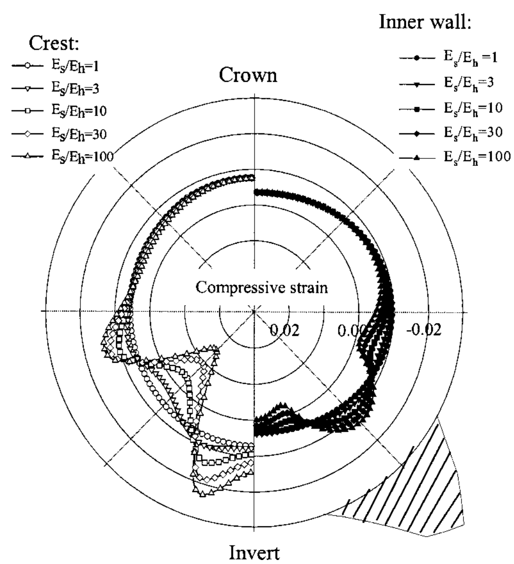

Fig. 15. Distribution of circumferential strains on polyvinyl chloride pipe

Table 2. Shape Factors Calculated for Low Stiffness Haunch Soil

		HDPE pipe		PVC pipe	
E_s/E_h		Valley	Crest	Inner wall	Crest
1	D_f	2.8	2.8	2.9	3.1
	θ	90°C	0°C/180°C	90°C	0°C/180°C
3	D_f	2.9	3.1	4.3	4.3
	θ	122°C	157°C	126°C	158°C
10	D_f	5.4	5.9	9.7	8.8
	θ	127°C	162°C	131°C	161°C
30	D_f	8.8	9.0	14.6	13.3
	θ	132°C	162°C	134°C	164°C
100	D_f	11.9	11.3	18.4	17.0
	θ	132°C	162°C	134°C	164°C

leads to strain redistribution around the pipe circumference, concentrating strains at the haunch. Variations of strains around the circumference are similar for the HDPE and the PVC pipes.

For pipes buried in uniform ground, circumferential strain on the valley or the inner surface at the springline represent the maximum value of compression the pipe encounters (as per classical concepts of buried pipe behavior). However, Figs. 14 and 15 show that the valley (inner surface for PVC pipe) compression decreases at the springline and increases at the haunch, with increases in E_s/E_h ratio. For the reduction of haunch soil stiffness by a factor of 100, the valley strain or inner wall strain between the ribs increases by 17% (from 2.9% at the springline to 3.4% near the mid haunch) on the HDPE pipe and by 38% (from 0.95% at the springline to 1.3% at the mid haunch) on the ribbed PVC pipe. The maximum compression is located near the middle of the low stiffness haunch region for both pipes, although the strain increase is much higher for the PVC pipe.

Strain on the crest of the profile increases even more as a result of the presence of a low stiffness soil under the haunch. At the invert, the crest strain is almost doubled for both the HDPE and PVC pipes when haunch soil modulus is reduced to 1% of the backfill soil. The strain increase is from 1.6 to 3.2% in the HDPE pipe and from 0.72 to 1.46% in the PVC pipe. Maximum compression on the crest is calculated near the boundary of the lower stiffness haunch soil. For E_s/E_h beyond 10, the maximum compression of the pipe section is on the crest at the haunch boundary. These large compressions may lead to local buckling or circumferential crushing of the elements.

The strain factors D_f for different values of E_s/E_h are calculated from the maximum bending strain obtained from the finite element method. Table 2 shows the factors along with the locations (θ) of the maximum bending strains, where θ is measured in degrees from the crown. The strain factors increase to about 12 for the HDPE pipe and to 18 for the PVC pipe as E_s/E_h increases to 100. This indicates that the influence of the low stiffness haunch soil is more significant for the PVC pipe. As E_s/E_h increases, the position of the maximum valley strain or inner wall strain moves from the springline towards the middle of the haunch, and the maximum crest strain moves from the invert to the boundary of the lower stiffness soil region under the haunch.

This analysis suggests possible values of strain factor that arise from a large zone of low stiffness material under the haunches. Final design recommendations would depend on an assessment of appropriate design conditions. Since specifications should always require care in placement of soil under the pipe haunches, the most severe conditions considered here are likely to be beyond typical design conditions. A typical range of elastic modulus in the haunch is expected to be from 5 to 20% of the backfill soil modulus.

Conclusion

Analysis of tests on buried HDPE and PVC pipes, reveals that the 2D finite element method can effectively be used to calculate both pipe deflections and circumferential strains. However, performance of the method relies on the appropriateness of the constitutive model used to characterize the soil behavior. Janbu's nonlinear soil model effectively simulated the influence of confining pressure on the soil modulus. Use in the analysis of a constant soil modulus was studied, with the best results obtained by averaging the stress dependent modulus over the expected stress range.

Calculation of pipe response by the finite element method matched the measurements within 12%. However, neither local bending nor local buckling within the profiled pipe wall can be considered in the two-dimensional analysis. However, the method can successfully be used to calculate the response of profile wall elements where three-dimensional effects are insignificant. Pipe deflections are not influenced by the local bending or local buckling, and local bending is not important for the ribbed PVC pipe.

While the finite element method provides a comprehensive design approach that can be used for high cost or unusual installations, the simplified design equations appeared as more suitable design tools for standard buried pipe installations. Structural stiffness of the pipes may have an influence on the performance of the simplified method. The method overestimated the deflections for the pipe with high hoop stiffness (the ribbed PVC test pipe) and underestimated for the pipe with low hoop stiffness (the corrugated HDPE test pipe) at 400 kPa overburden pressure.

Strain factor used in the simplified equations, D_f, can be calculated from the maximum bending strain obtained from the finite element analyses of the tests. The estimated values for uniform ground support conditions are close to 3, the theoretical value for ''classical'' elliptical pipe deformation. However, nonuniformity of the surrounding soil support has a significant effect on the behavior of buried pipe. A zone of low stiffness soil placed under the haunch changes the distribution of strain around the pipe circumference. The strains concentrate on the inner surface of the pipe (under the valley of lined corrugated HDPE pipe) towards the middle of the haunch zone, and on the crest at the boundary of the zone of low stiffness haunch soil. For the cases considered, the crest strain appeared to govern the design when the stiffness of the haunch soil was less than one tenth of the stiffness of the surrounding backfill. Haunch support appeared to affect the ribbed PVC pipe strains more significantly than those for the corrugated HDPE pipe. Strain factor values of approximately 9 and 6 were obtained, respectively, for PVC and HDPE pipes with soil modulus ratio $E_s/E_h=10$ (corresponding to about a 15% reduction in compaction density under the haunch). Strain factor values rose to over 18 and 12 for PVC and HDPE pipes where $E_s/E_h=100$.

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