

Evaluation of Simplified Design Methods for Buried Thermoplastic Pipe

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Abstract

Simplified equations to calculate deflections and circumferential strains in buried thermoplastic pipe have been developed for AASHTO based on soil-structure interaction solutions. These equations are evaluated here using measurements of HDPE and PVC pipe responses in full-scale laboratory tests. The comparison indicates that the equations are suitable design tools for standard flexible pipe installations. Structural stiffness of the pipes may have an influence on the performance of the deflection equation, since the method overestimated the deflections for a ribbed PVC pipe with high hoop stiffness. The methods for estimating hoop strain and vertical arching factor worked very well for both pipes, and the equation for bending strains provided somewhat conservative values. Recommended design values for constrained modulus of the soil lead to conservative but reasonable predictions in all cases.

Introduction

A limit states design procedure based on simplified equations is being developed for design of thermoplastic pipes by engineers using the AASHTO bridge design code. While the two-dimensional elastic continuum theory for pipe-soil interaction (Hoeg, 1968) might be adapted for design of various different buried pipes, Moore (2000), the equations associated with that theory are likely too complicated for use in codified design procedures, and are currently unable to assess the impact of variable installation conditions such as poor haunch support. Therefore, simplified design equations have been proposed specifically for thermoplastic pipes. McGrath (1998a) proposed simplified design equations to calculate pipe deflections and wall strains based on the continuum approach (Burns and Richard, 1964), finite element analyses, and field observations. The performance of those equations is evaluated here, using measurements of pipe deflections and wall strains in three full-scale tests in a controlled laboratory environment. Responses of pipes under biaxial loading were measured during biaxial tests performed in a biaxial pipe test cell (Dhar, 2002). The tests simulate biaxial field stress conditions in the laboratory. Two lined-corrugated, high-density polyethylene pipes and a ribbed polyvinyl chloride pipe were tested. Figure 1 shows the wall profiles of those three pipes and the locations of strain measurements. Sectional properties for those profiles are described in Table 1. Stresses and deformations of the backfill soil were also measured during each of the tests. Two sets of soil parameters are used in alternative analyses of these test results. First, "measured" values of

constrained soil modulus were calculated using measurements of vertical strain and vertical stress in the soil during the tests, Dhar et al (2002). Second, the design values of constrained soil response proposed by McGrath (1998b) for use in buried pipe design were examined. Design values of backfill soil modulus were proposed for three general classes of soil (SW, ML and CL) at different levels of vertical stress. The backfill material used in the tests is classified as an SP85, which is a part of the SW class.

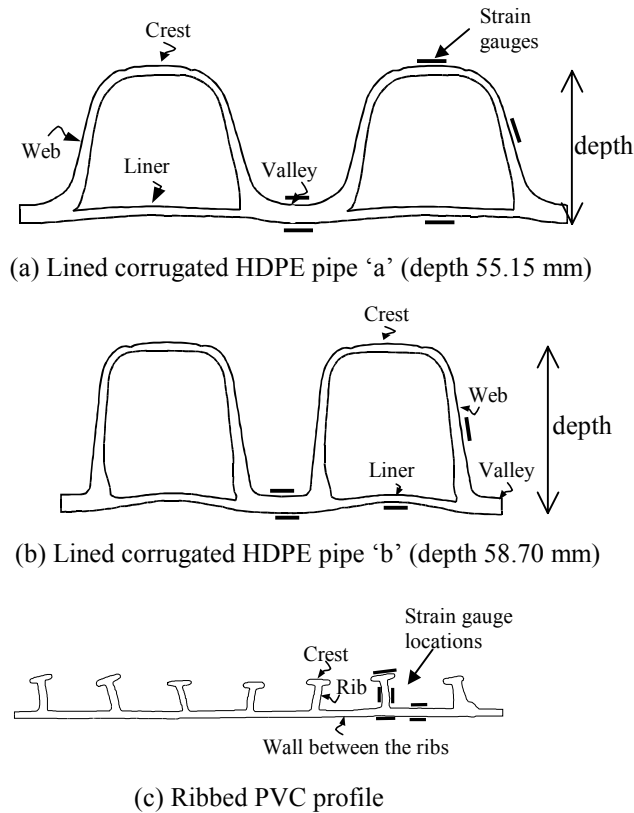


Figure 1: Wall profiles of the pipes

Table 1. Section properties for the pipe profiles

| Test pipe | Modulus (MPa) | Poisson's ratio, ν | EA (N/mm) | EI (N-mm ² /mm) |
|-----------|---------------|------------------------|-----------|----------------------------|
| HDPE 'a' | 450 | 0.46 | 4187 | 1.40×10^6 |
| HDPE 'b' | 450 | 0.46 | 4562 | 1.79×10^6 |
| PVC | 2760 | 0.30 | 19480 | 1.14×10^6 |

Simplified Equation for Pipe Deflection

The simplified equation for pipe deflection (McGrath, 1998a) is expressed in a form similar to the Iowa equation (Spangler, 1941); however, it incorporates contributions to vertical diameter decrease from the hoop compression as well as the flexure of the pipe-soil system:

$$\frac{\Delta_v}{D} = \left(\frac{VAF \cdot q_v}{\frac{EA}{R} + 0.57M_s} \right) + \left(\frac{D_1 K_b q_v}{\frac{EI}{R^3} + 0.061M_s} \right) \quad (1)$$

The first term in Equation 1 represents the average circumferential shortening, while the second is the flexural term similar to one Spangler (1941) proposed for quantifying bending deformations. However, the constrained modulus, M_s is used instead of the semi-empirical soil stiffness parameter, the modulus of soil reaction, E' employed by Spangler, McGrath (1998b). Other parameters in the equation are defined as follows:

- D_v = Decrease in vertical pipe diameter (mm, in.)
- D = Pipe diameter (mm, in.)
- q_v = Overburden pressure at the springline (MPa, psi)
- E = Pipe material modulus (MPa, psi)
- R = Radius of the centroid of the pipe section (mm, in.)
- M_s = One dimensional soil modulus (MPa, psi)
- K_b = Bedding coefficient
- D_1 = Deflection lag factor
- VAF = Vertical Arching Factor

The VAF is included in only the left hand term of Equation 1 since McGrath (1998a) suggested use of the full overburden stress for the flexural (right hand) deflection term. Equation 1 can also be used to calculate horizontal diameter change by subtracting the second term of the equation from the first term.

Equations for Wall Strains

Introduction. The circumferential wall strain for a buried pipe is composed of its hoop component, ϵ_h and bending component, ϵ_b , so that, $\epsilon = \epsilon_h + \epsilon_b$. Simplified equations can be developed to calculate these two components of strains separately.

Hoop Strain. Hoop strain at any pipe section can be calculated from thrust acting on the section. For thrust per unit length N , the hoop strain at any section is given by,

$$\left. \begin{aligned} \epsilon_h &= \frac{N}{EA} && \text{for a plane stress condition} \\ \epsilon_h &= \frac{(1-\nu^2)N}{EA} && \text{for a plane strain condition} \end{aligned} \right\} \quad (2)$$

Here,

- N = hoop thrust per unit length (N/mm, pounds/in.)
- EA = hoop stiffness per unit length (N/mm, pounds/in.)
- ν = Poisson's ratio of the pipe material.

The hoop thrust is non-uniform around the circumference for pipes in biaxially stressed ground (lateral earth pressure coefficient, $K \neq 1$). McGrath (1998a) proposed a simplified

equation to calculate the maximum thrust that occurs at the springline. The equation employs the Vertical Arching Factor (VAF) to account for the interaction of the pipe with the surrounding soil:

$$N = q_v R_o (\text{VAF}) \quad (3)$$

Here, N = The maximum springline thrust per unit length (N/mm, pounds/in.)
 q_v = Vertical geostatic soil stress (MPa, psi)
 R_o = Outside pipe radius (mm, psi)

The vertical arching factor (VAF) in Equation 3 gives the proportion of the weight of the soil column over the pipe (the prism load) that reaches the pipe. McGrath (1998a) proposed an approximate design solution for the VAF on the basis of his study using the fundamental elasticity solution of Burns and Richard (1964):

$$\text{VAF} = 0.76 - 0.71 \left(\frac{S_H - 1.17}{S_H + 2.92} \right) \quad (4)$$

where, S_H = Hoop Stiffness Factor = $\frac{M_s R}{EA}$ and M_s is the constrained modulus of the soil (MPa, psi). Equation 4 was adopted by AASHTO in 2000.

Bending strain. A semi-empirical formula can be used to calculate the peak circumferential bending strain on a deformed pipe (Watkins, et al., 1973):

$$\epsilon_b = D_f \left(\frac{c}{R} \right) \left(\frac{\Delta_b}{D} \right) \quad (5)$$

where c is the distance to the extreme fiber from the neutral axis (mm, in.), D is the pipe diameter (mm, in.), R is radius (mm, in.) and Δ_b is the bending deflection (mm, in.).

Equation 5 relates the bending deflection (Δ_b) of the pipe to the peak circumferential bending strain through an empirical strain factor D_f . The bending component of the pipe deflection can be obtained from the second part of Equation 1, i.e.,

$$\frac{\Delta_b}{D} = \frac{D_l K_b q_v}{\frac{EI}{R^3} + 0.061 M_s} \quad (6)$$

The magnitude of the strain factor is 3 for pure bending of a thin circular incompressible ring when deformed into an elliptical shape (Roark, 1943). The maximum bending strain is located at the springline for the elliptically deformed ring. Elliptical deformation is expected for the buried pipe when the backfill soil is uniform. However, if the backfill soil is non-uniform, the pipe may deflect to a shape different from the elliptical. As a result, the maximum bending strain can be located away from the springline, depending upon the conditions of ground support. The strain factor, D_f , for the maximum bending strain will also be different.

Evaluation of the Simplified Equations

Introduction. In this section, the simplified design equations are evaluated using the measurements and the quantities calculated from finite element analysis. Dhar, et al. (2002) employed two-dimensional finite element analyses to examine the behavior of the pipes in the test cell. Measured pipe deflections and the circumferential strains are compared here with those obtained using the simplified design equations and the finite element analyses. Calculations using the continuum solution of Hoeg (1968) are also included in the comparison.

As indicated earlier, the proposed design equations are used with both the measured and the design soil parameters. The comparison of measured strain or deflection with values calculated using measured soil modulus provides information about the level of approximation in the pipe-soil interaction theory being used. A comparison of measured strain or deflection with values calculated using design soil modulus provides information about the overall level of approximation associated with the proposed design calculation for these particular pipe burial conditions (soil type, placement procedure, burial depth).

Pipe Deflections. The simplified design equation for deflection (Equation 1) involves a bedding factor and a constrained modulus for the soil. Spanger and Handy (1973) recommended the values of the bedding factor for various bedding angles from 0° (no bedding) to 180° (bedding up to the springline). A factor of 0.083 is used in this study as suggested for 180° bedding (Spangler and Handy, 1973), since backfill in the test cell was placed with care to provide good support to the pipe. The secant value of the constrained modulus, M_s , was used for one set of calculations and the average value of constrained modulus computed using linear regression from the measured stress-strain relations (Dhar et al., 2002) was employed for a second set of calculations of deflection. Table 2 shows the two sets of constrained moduli and the corresponding elastic moduli at vertical stresses of 100 kPa and 400 kPa. The secant and the average modulus are the same for the test with HDPE pipe 'b' because the stress-strain response was essentially linear for that test (Dhar, et al., 2002).

Table 2. Soil modulus at vertical stresses of 100 kPa and 400 kPa

| Stress kPa | Test pipe | Constrained modulus, M_s , MPa | | Elastic modulus, E, MPa | | Design constrained modulus, MPa |
|---------------|--------------|--|---------|----------------------------|---------|--|
| | | Secant | Average | Secant | Average | |
| 100 | HDPE 1 | 6.8 | 5.1 | 4.6 | 3.4 | 4.2 |
| | HDPE 2 | 11.3 | 11.3 | 6.4 | 6.4 | |
| | PVC | 5.7 | 5.5 | 3.9 | 3.7 | |
| 400 | HDPE 1 | 11.5 | 9.6 | 7.7 | 6.5 | 6.8 |
| | HDPE 2 | 11.3 | 11.3 | 6.4 | 6.4 | |
| | PVC | 10.3 | 8.7 | 6.9 | 5.9 | |

Table 3 presents the calculated values of pipe deflection obtained using various methods both at vertical earth pressures of 100 kPa and 400 kPa. The continuum theory gave greater deflection values than the simplified method for the lined corrugated HDPE pipes, and lower deflection for the ribbed PVC pipe.

The calculation with secant modulus provides estimates of the deflections that are sometimes unconservative. However, when the average modulus was used, the simplified

equation provided higher deflections than the measurements (except for HDPE pipe ‘b’). The level of error for the two HDPE pipes increased at 400 kPa, reaching 13% for HDPE pipe ‘b’. This is caused by geometrical non-linearity, which is not captured by the simplified equation. The non-linear material response of the soil is captured reasonably by use of the secant or average modulus, and the calculated deflections match the measurements particularly well at the lower stress level (i.e. 100 kPa). The simplified equation with design values of soil modulus provides conservative estimates of deflection for all of the pipes.

Deflections calculations based on the non-linear finite element analyses using measured soil properties described by Dhar, et al. (2002) are also included in Table 3, and generally provide the best estimates of pipe deflection. The finite element analysis accounts for the soil non-linearity that controls the behavior of the pipes in the biaxial cell. Naturally, the level of effort associated with undertaking a nonlinear finite element calculation is considerable, and the simplified deflection equation is considered suitable for obtaining reasonable design estimates of pipe deflection.

Table 3. Change in vertical diameter (mm) due to bending at 100 kPa and 400 kPa

| Stress kPa | Test pipes | Expt. | FE | Using secant modulus | | Using Avg. modulus | | Using design modulus | |
|------------|------------|-------|-------|----------------------|-------|--------------------|-------|----------------------|-------|
| | | | | (a)* | (b)** | (a)* | (b)** | (a)* | (b)** |
| 100 | HDPE ‘a’ | -16.8 | -16.9 | -15.6 | -16.1 | -19.3 | -20.0 | -22.3 | -22.8 |
| | HDPE ‘b’ | -9.9 | -10.5 | -10.5 | -11.1 | -10.5 | -11.1 | -21.6 | -19.2 |
| | PVC | -13.1 | -13.1 | -14.2 | -12.9 | -14.7 | -13.4 | -18.4 | -16.7 |
| 400 | HDPE ‘a’ | -47.2 | -48.1 | -42.2 | -44.6 | -48.3 | -50.4 | -62.4 | -64.4 |
| | HDPE ‘b’ | -48.8 | -45.1 | -42.0 | -44.3 | -42.0 | -44.3 | -61.1 | -55.5 |
| | PVC | -34.6 | -34.7 | -34.4 | -31.7 | -39.7 | -36.3 | -49.0 | -44.7 |

* (a) Simplified method (McGrath 1998a)

** (b) Continuum theory (Hoeg 1968): Bonded interface

Vertical Arching Factor (VAF). Table 4 compares the Vertical Arching Factors obtained using various methods to those obtained from interpretation of the three buried pipe tests. The VAF was calculated using Equation 4, employing both the measured and the design soil parameters. VAF results obtained from finite element analysis are also included, Dhar, et al. (2002). The experimental values of VAF have been calculated from measurements of hoop strain, and the relation $VAF = EA\varepsilon_h/PR_o$.

The finite element method consistently provides upper bound values of the VAF. These finite element calculations are approximately 10% higher than those obtained from the simplified equation. The simplified equation with design soil modulus (McGrath, et al. 1999) also provides consistently high values of the factor. In all cases, the “measured” values of VAF are less than the calculated values.

All the arching factors for the HDPE pipes were less than unity. The particularly low measured value for HDPE pipe ‘b’ at 100-kPa is due to the fact that the test had to be re-run; the air bladder used in that test failed at a cell pressure of 75 kPa and the same test was reloaded using a new bladder. This led to a higher value of initial soil modulus and hence a lower VAF. Equation (4) with measured soil modulus provides VAF close to the other measured values for the HDPE pipes (Table 4).

For PVC pipe, the calculated value of VAF is greater than 1, indicating that the pipe is stiffer than the surrounding soil. Equation 4 provides conservative but reasonable values of VAF compared to measurements for the PVC pipe.

Based on these comparisons, Equation 4 provides reasonable design calculations of VAF for both these thermoplastic pipes, and appears to be a suitable tool for use in culvert design.

Table 4. Vertical arching factors

| Tests | Stress level | Constrained modulus, MPa | | FE using measured modulus | Simplified Method | | Measured |
|-------------|--------------|--------------------------|---------------------|---------------------------|--------------------------|------------------------|----------|
| | | M _{measured} | M _{design} | | (M _{Measured}) | (M _{design}) | |
| HDPE 'a' | 100 | 6.87 | 4.17 | 0.98 | 0.89 | 0.95 | 0.84 |
| | 200 | 8.88 | 5.02 | 0.94 | 0.86 | 0.93 | 0.83 |
| | 300 | 10.31 | 5.91 | 0.92 | 0.83 | 0.91 | 0.84 |
| | 400 | 11.47 | 6.81 | 0.90 | 0.81 | 0.89 | 0.86 |
| HDPE 'b' | 100 | 11.3 | 4.17 | 0.89 | 0.83 | 0.95 | 0.65 |
| | 200 | 11.3 | 5.02 | 0.89 | 0.83 | 0.94 | 0.82 |
| | 300 | 11.3 | 5.91 | 0.89 | 0.83 | 0.92 | 0.83 |
| | 400 | 11.3 | 6.81 | 0.89 | 0.83 | 0.90 | 0.84 |
| PVC | 100 | 5.28 | 4.17 | 1.17 | 1.02 | 1.02 | 0.80 |
| | 200 | 7.42 | 5.02 | 1.16 | 1.01 | 1.02 | 0.85 |
| | 300 | 8.97 | 5.91 | 1.15 | 1.00 | 1.01 | 0.90 |
| | 400 | 10.25 | 6.81 | 1.14 | 0.99 | 1.01 | 0.92 |

Strains. Hoop strains and bending strains are calculated separately using both the simplified model and the finite element method for comparison with strain measurements. Hoop strains from the measurements are estimated as the values at the centroid axis of the profile sections, assuming a linear best-fit distribution of strains through the profile. Hoop strains are then subtracted from the total strain to obtain the bending strains. Figure 2 illustrates how the hoop and bending strains are estimated from one particular set of strain measurements. Strains at the springline of the pipe are considered, since the springline strain on the interior pipe surface corresponds to one of the peak values of wall compression in biaxially loaded pipes. Since strain at any point is the sum of the hoop and bending strains, it is sufficient to examine values at one specific location; springline strains on the interior surface of pipes are chosen for the comparison.

Equation 6 is used to calculate the bending component of deflection for use in the bending strain equation (Equation 5). Values of the empirical factors in Equation 6 were, $D_1 = 1$ and $K_b = 0.083$. A strain factor of 3 was used in Equation 5 assuming an elliptical deformation shape for the pipe in the test cell. Measured values of constrained modulus were used in the simplified equations to calculate the pipe strains.

Figures 3(a) and 3(b) compare the measurements of hoop and bending strains with the design estimates for HDPE pipe 'a'. The finite element method overestimated the hoop strain by about 15% for this profile (Figure 3a). However, the finite element value of bending strain matches the measurements well (Figure 3b). Finite element estimates of hoop strains also exceed the measured values for the other profiles (Figures 4 and 5), while the bending strains were

calculated reasonably. The assumption in the finite element analysis of a bonded interface between the pipe and the surrounding soil may have caused this high calculated hoop strain, since this increases calculated thrusts at the springline. The high hoop strains in the finite element calculations may also result from a plane stress idealization of the pipe in the analysis, which neglects the restraint provided by axial stiffness to the circumferential strains (Dhar et al., 2002). The VAFs calculated by the finite element method seen earlier in Table 4 were correspondingly high.

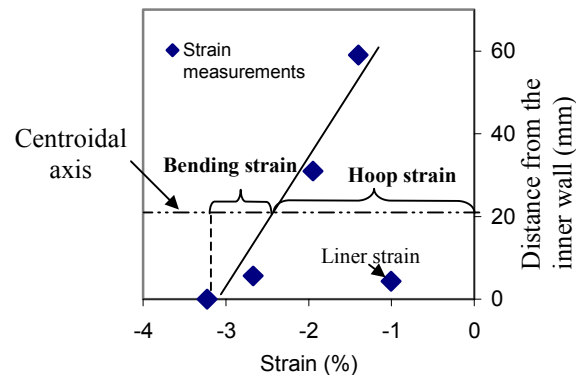


Figure 2: Calculation of strain components from measurements

The simplified equations for hoop and bending strains also appear to work well for the other pipes (Figures 4 and 5). The equations provided upper bound values of the strains on the PVC pipe. Neglect of the Poisson's effect (restraint provided by axial stiffness) in the calculation might have contributed to this overestimation. Large values of bending strains on the second HDPE pipe were also obtained using the simplified method. Performance of the equation for bending strain depends on choice of parameters like, D_f and K_b . The D_f value of 3 used in the calculation is applicable for an elliptical deformation shape, an idealized pattern that will not be achieved in the field. For non-elliptical deformation, the strain may no longer reach a peak at the springline. The simplified models with design soil parameters (McGrath, et al., 1999) furnished conservative estimates (greater values) of strains, since the design soil moduli are less than the measured values

Conclusions

Overall, this study supports the use of the simplified design equations proposed by McGrath (1998a) in design of standard thermoplastic pipe installations.

1. The simplified equation for deflection incorporates both the hoop and bending components of the pipe-soil response, where the first of these terms is important for compressible pipe such as thermoplastic culverts. The design equation generally worked effectively for the HDPE and PVC pipes considered here.
2. Deflection predictions based on "average" soil modulus were somewhat superior to those obtained using "secant" modulus, but this improvement is not considered sufficient to warrant a move away from the use of secant soil modulus in culvert design, which is becoming the normal practice in North America.

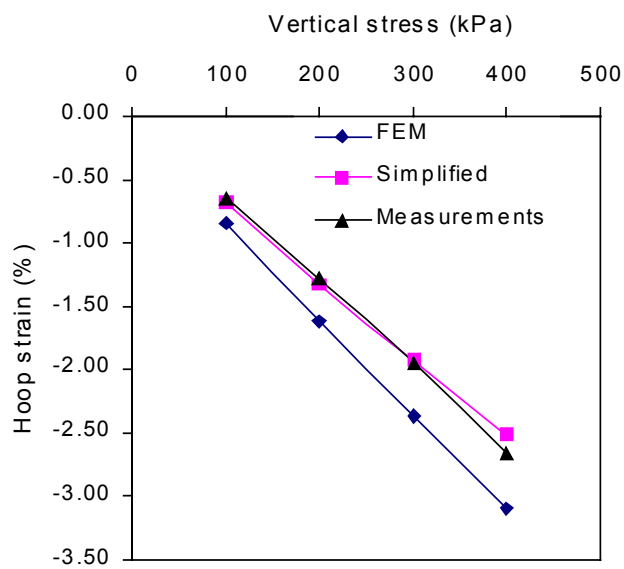
3. The equations proposed for calculating hoop and bending strains provided good estimates of the circumferential strains at the springline of the test pipes. These tests involved good quality soil support below the haunches of the pipe, and values of strain factor D_f for field conditions should be increased to account for these effects, Dhar (2002). Ongoing work is considering this further.
4. The equation for Vertical Arching Factor provides a reasonable estimate of the effect of pipe-soil interaction on the thrust that develops at the pipe springline.

Acknowledgements

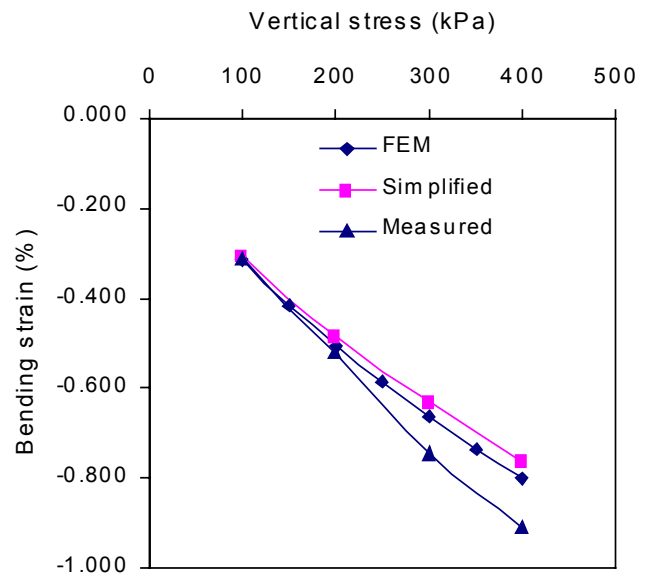
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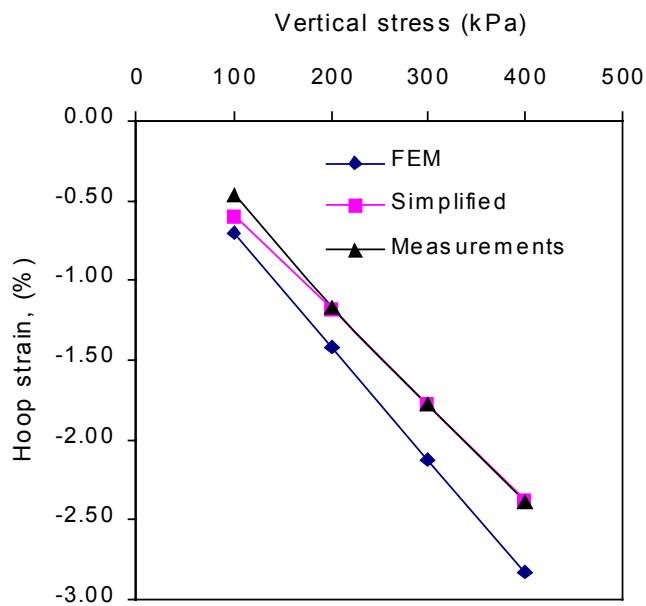


(a) Hoop Strain

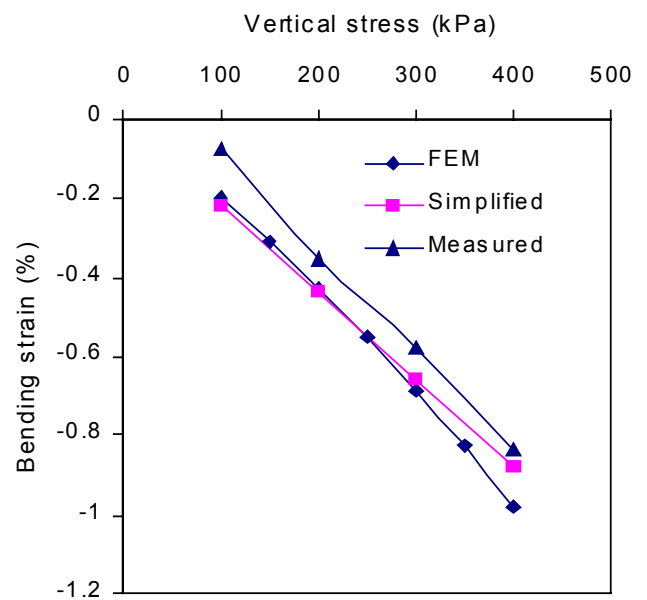


(b) Bending Strain

Figure 3: Comparison of strains on the interior surface at the springline of the HDPE pipe 'a'

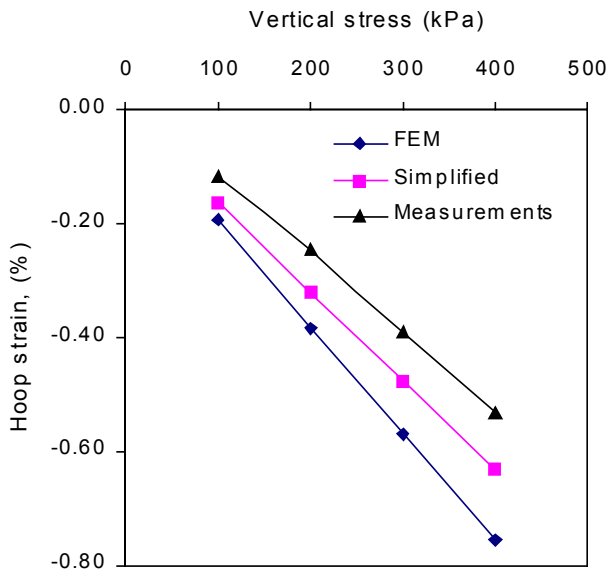


(a) Hoop Strain

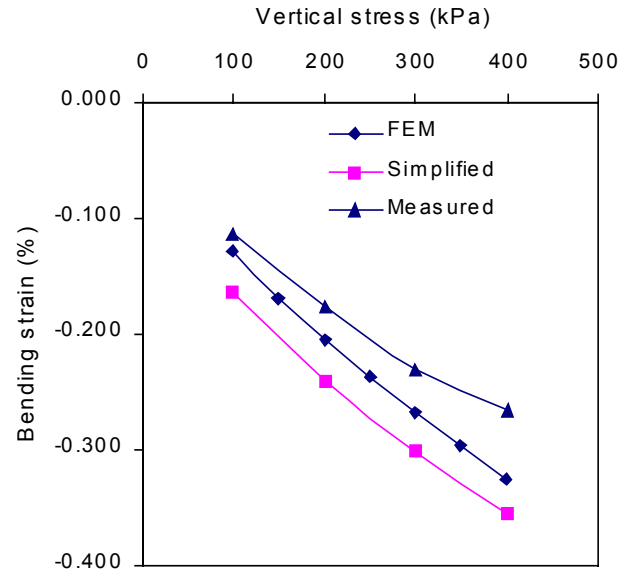


(b) Bending Strain

Figure 4: Comparison of strains on the interior surface at the springline of the HDPE pipe 'b'



(a) Hoop Strain



(b) Bending Strain

Figure 5: Hoop and bending strain comparison for the PVC pipe
(on the interior surface at the springline)